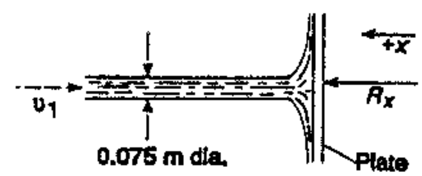
CHAPTER SIXTEEN

FORCES DUE TO FLUIDS IN MOTION

16.1 $Q = A\upsilon = \frac{\pi(0.075\text{ m})^2}{4} \times \frac{25\text{ m}}{\text{s}} = \frac{0.1104\text{ m}^3}{\text{s}}$



$R_x = \rho Q(\upsilon_{2_x} - \upsilon_{1_x}) = \rho Q(0 - (-\upsilon_1)) = \rho Q\upsilon_1$

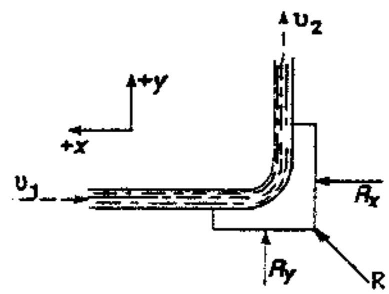
$R_x = \frac{1000\text{ kg}}{\text{m}^3} \times \frac{0.1104\text{ m}^3}{\text{s}} \times \frac{25\text{ m}}{\text{s}} = \frac{2761\text{ kg}\cdot\text{m}}{\text{s}^2}$

$= 2761\text{ N} = \mathbf{2.76\text{ kN}}$

16.2 See sketch for Problem 16.1: $R_x = \rho Q\upsilon_1 = \rho(A\upsilon_1)\upsilon_1 = \rho A\upsilon_1^2$

$\upsilon = \sqrt{\frac{R_x}{\rho A}} = \sqrt{\frac{300\text{ lb}\ \ 144\text{ in}^2/\text{ft}^2}{(1.94\text{ lb-s}^2/\text{ft}^4)(\pi(2.0\text{ in})^2/4)}} = \mathbf{84.2\text{ ft/s}}$

16.3 $Q = 150\text{ gal/min} \times \frac{1\text{ ft}^3/\text{s}}{449\text{ gal/min}} = 0.334\text{ ft}^3/\text{s}$



$\upsilon_1 = \upsilon_2 = \frac{Q}{A} = \frac{0.324\text{ ft}^3/\text{s}}{\pi(1/12\text{ ft})^2/4} = 61.25\text{ ft/s}$

$R_x = \rho Q(\upsilon_{2_x} - \upsilon_{1_x}) = \rho Q(0 - (-\upsilon_1)) = \rho Q\upsilon_1$

$R_x = \frac{1.94\text{ lb-s}^2}{\text{ft}^4} \times \frac{0.334\text{ ft}^3}{\text{s}} \times \frac{61.25\text{ ft}}{\text{s}} = \mathbf{39.7\text{ lb}}$

$R_y = \rho Q(\upsilon_{2_y} - \upsilon_{1_y}) = \rho Q(\upsilon_2 - 0) = \rho Q\upsilon_2 = \rho Q\upsilon_1 = \mathbf{39.7\text{ lb}}$

16.4 Assume all air leaves parallel to the face of the sign.

$\upsilon_1 = 125\text{ km/h} \times \frac{10^3\text{ m}}{\text{km}} \times \frac{1\text{ h}}{3600\text{ s}} = 34.7\text{ m/s}$

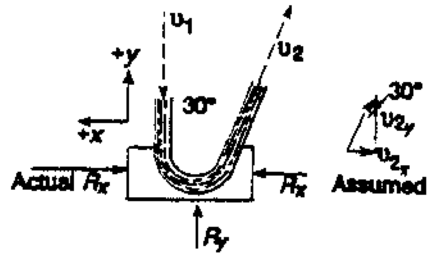
$R_x = \rho Q(\upsilon_{2_x} - \upsilon_{1_x}) = \rho Q(0 - (-\upsilon_1)) = \rho Q\upsilon_1 = \rho(A\upsilon_1)(\upsilon_1) = \rho A\upsilon_1^2$

$R_x = \frac{1.341\text{ kg}}{\text{m}^3} \times (3\text{ m})(4\text{ m}) \times \frac{(34.7)^2\text{ m}^2}{\text{s}^2} = \frac{19.4\times10^3\text{ kg}\cdot\text{m}}{\text{s}^2} = \mathbf{19.4\text{ kN}}$

Equivalent pressure $= p = \frac{R_x}{A} = \frac{19.4\times10^3\text{ N}}{12\text{ m}^2} = \frac{1617\text{ N}}{\text{m}^2} = \mathbf{1617\text{ Pa}}$

16.5 $Q = A\upsilon = \dfrac{\pi(1.75\text{ in})^2}{4} \times \dfrac{\text{ft}^2}{144\text{ in}^2} \times \dfrac{25\text{ ft}}{\text{s}}$

$= 0.418\text{ ft}^3\text{/s}$



$R_x = \rho Q(\upsilon_{2_x} - \upsilon_{1_x}) = \rho Q[-\upsilon_2 \sin 30° - (0)]$

$R_x = -\rho Q\upsilon_2 \sin 30° = -(1.94)(0.418)(25) \sin 30°$

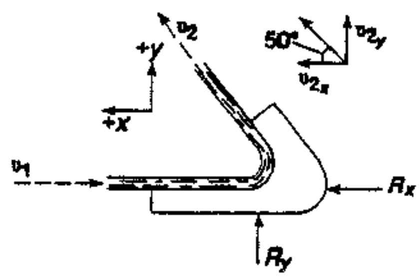
$R_x = -10.13$ lb = **10.13 lb to right**

$R_y = \rho Q(\upsilon_{2_y} - \upsilon_{1_y}) = \rho Q[\upsilon_2 \cos 30° - (-\upsilon_1)]$

$R_y = \rho Q[\upsilon_2 \cos 30° + \upsilon_1] = (1.94)(0.418)[25 \cos 30° + 25] =$ **37.79 lb up**

16.6 $Q = A\upsilon = (2.95\text{ in}^2)(22.0\text{ ft/s})(1\text{ ft}^2/144\text{ in}^2)$

$= 0.451\text{ ft}^3\text{/s}$



$R_x = \rho Q(\upsilon_{2_x} - \upsilon_{1_x}) = \rho Q[\upsilon_2 \cos 50° - (-\upsilon_1)]$

$R_x = \rho Q[\upsilon_2 \cos 50° + \upsilon_1]$; but $\upsilon_2 = \upsilon_1$

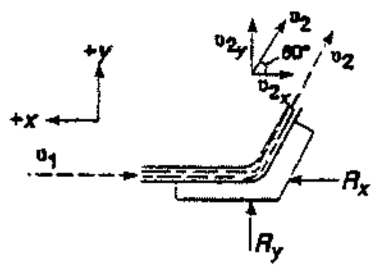
$R_x = \rho Q\upsilon_1[\cos 50° + 1]$

$R_x = \dfrac{1.88\text{ lb}\cdot\text{s}^2}{\text{ft}^4} \times \dfrac{0.451\text{ ft}^3}{\text{s}} \times \dfrac{22.0\text{ ft}}{\text{s}} \times 1.643$

$=$ **30.6 lb**

$R_y = \rho Q(\upsilon_{2_y} - \upsilon_{1_y}) = \rho Q[\upsilon_2 \sin 50° - 0] = (1.88)(0.451)(22 \sin 50°) =$ **14.3 lb**

16.7 $Q = A\upsilon = \dfrac{\pi(0.10\text{ m})^2}{4} \times \dfrac{15\text{ m}}{\text{s}} = 0.118\text{ m}^3\text{/s}$



$R_x = \rho Q[\upsilon_{2_x} - \upsilon_{1_x}] = \rho Q[-\upsilon_2 \cos 60° - (-\upsilon_1)]$

$R_x = \rho Q\upsilon_1[1 - \cos 60°]$

$R_x = \dfrac{988\text{ kg}}{\text{m}^3} \times \dfrac{0.118\text{ m}^3}{\text{s}} \times \dfrac{15\text{ m}}{\text{s}} \times [0.5]$

$= \dfrac{873\text{ kg}\cdot\text{m}}{\text{s}^2} =$ **873 N**

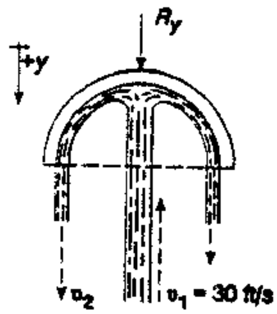
$R_y = \rho Q[\upsilon_{2_y} - \upsilon_{1_y}] = \rho Q[\upsilon_2 \sin 60° - 0] = (988)(0.118)(15)(\sin 60°) =$ **1512 N**

16.8 $Q = A_1 \upsilon_1 = \dfrac{\pi(1.00\text{ in})^2}{4} \times \dfrac{\text{ft}^2}{144\text{ in}^2} \times \dfrac{30\text{ ft}}{\text{s}} = \dfrac{0.1636\text{ ft}^3}{\text{s}}$

$\upsilon_2 = \dfrac{Q}{A_2} = \dfrac{0.1636\text{ ft}^3}{\text{s}} \dfrac{144\text{ in}^2/\text{ft}^2}{\pi(4.00^2 - 3.80^2)\text{ in}^2/4} = \dfrac{19.23\text{ ft}}{\text{s}}$

$R_y = \rho Q\left(\upsilon_{2_y} - \upsilon_{1_y}\right)$

$R_y = \dfrac{1.88\text{ lb-s}^2}{\text{ft}^4} \times \dfrac{0.1636\text{ ft}^3}{\text{s}}[19.23 - (-30)]\dfrac{\text{ft}}{\text{s}} = \mathbf{15.14\ lb}$

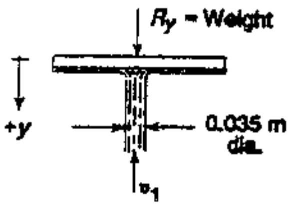


16.9 $R_y = 550\text{ N} = 550\text{ kg·m/s}^2 = \rho Q\left(\upsilon_{2_y} - \upsilon_{1_y}\right)$

$= \rho A_1 \upsilon_1(0 - (-\upsilon_1)) = \rho A_1 \upsilon_1^2$

$\upsilon_1 = \sqrt{\dfrac{R_y}{\rho A_1}} = \sqrt{\dfrac{550\text{ kg·m/s}^2}{(900\text{ kg/m}^3)(9.62\times10^{-4}\text{ m}^2)}} = \mathbf{25.2\ m/s}$

$A_1 = \dfrac{\pi(0.035\text{ m})^2}{4} = 9.62\times10^{-4}\text{ m}^2$



16.10 $R_{x_1} = \rho Q\left(\upsilon_{2_x} - \upsilon_{1_x}\right) = \rho A \upsilon_1[0 - (-\upsilon_1)] = \rho A_1 \upsilon_1^2$

$A_1 = \dfrac{1}{2}\left[\dfrac{\pi(2.0\text{ in})^2}{4} \times \dfrac{\text{ft}^2}{144\text{ in}^2}\right] = 0.0109\text{ ft}^2$

$R_{x_1} = \dfrac{1.94\text{ lb·s}^2}{\text{ft}^4} \times 0.0109\text{ ft}^2 \times \left(\dfrac{40.0\text{ ft}}{\text{s}}\right)^2$

$= \mathbf{33.9\ lb}$

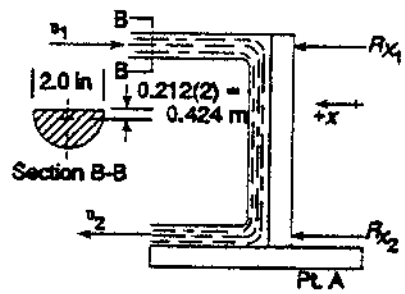
$R_{x_2} = \rho Q\left(\upsilon_{2_x} - \upsilon_{1_x}\right) = \rho Q(\upsilon_2 - 0) = \rho Q \upsilon_2$

Assume $\upsilon_2 = \upsilon_1$, $A_2 = A_1$

$R_{x_2} = \rho A \upsilon_1^2 = \mathbf{33.9\ lb}$

$M_{1_A} = R_{x_1}(4.00 - 0.424)\text{in} = (33.9\text{ lb})(3.576\text{ in}) = \mathbf{121\ lb{\cdot}in}$

Moment due to R_{x_2} is small–depends on shape of leaving stream.



16.11 $Q = 100\text{ gal/min} \times \dfrac{1\text{ ft}^3/\text{s}}{449\text{ gal/min}} = 0.223\text{ ft}^3/\text{s}$: $\upsilon_1 = \dfrac{Q}{A} = \dfrac{0.223\text{ ft}^3/\text{s}}{0.0060\text{ ft}^2} = 37.1\text{ ft/s}$

Assume all fluid strikes vane and is deflected perpendicular to incoming stream.

$R_x = \rho Q\left(\upsilon_{2_x} - \upsilon_{1_x}\right) = \rho Q[0 - (-\upsilon_1)] = \rho Q \upsilon_1$

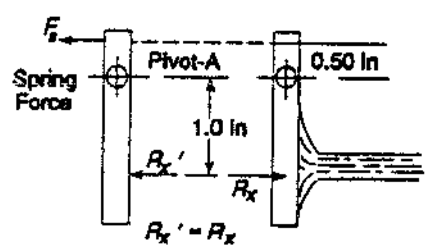
$R_x = \dfrac{1.94\text{ lb·s}^2}{\text{ft}^4} \times \dfrac{0.223\text{ ft}^3}{\text{s}} \times \dfrac{37.1\text{ ft}}{\text{s}} = 16.0\text{ lb}$

R_x is force exerted by vane on water

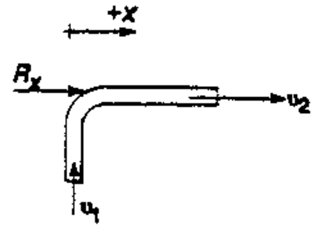
R_x' is force exerted by water on vane

$\Sigma M_A = 0 = F_S(0.5\text{ in}) - R_x'(1.0\text{ in})$

$F_S = R_x' \dfrac{1.0}{0.5} = 2R_x' = 2(16.0\text{ lb}) = \mathbf{32.0\ lb}$



16.12 $R_x = \rho Q[\upsilon_{2_x} - \upsilon_{1_x}] = \rho A\upsilon_2[\upsilon_2 - 0] = \rho A\upsilon_2^2$



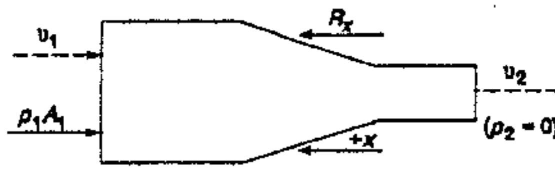
$$A = \frac{\pi(4.0\text{ in})^2}{4} \times \frac{\text{ft}^2}{144\text{ in}^2} = 0.0873\text{ ft}^2$$

$$R_x = \frac{1.94\text{ lb}\cdot\text{s}^2}{\text{ft}^4} \times 0.0873\text{ ft}^2 \times (60\text{ ft/s})^2$$

= 609 lb acting on water jet

Force on boat is reaction to R_x acting toward left. ←

16.13 $$\frac{p_1}{\gamma} + \frac{\upsilon_1^2}{2g} - h_L = \frac{p_2}{\gamma} + \frac{\upsilon_2^2}{2g}$$



$$\upsilon_1 = \upsilon_2 \cdot \frac{A_2}{A_1} = 80 \cdot \frac{0.0218}{0.0873}$$

= 20 ft/sec

$$p_1 = \gamma\left[\frac{\upsilon_2^2}{2g} - \frac{\upsilon_1^2}{2g} + h_L\right] = \gamma\left[\frac{\upsilon_2^2}{2g} - \frac{\upsilon_1^2}{2g} + \frac{0.12\upsilon_2^2}{2g}\right] = \gamma\left[\frac{1.12\upsilon_2^2 - \upsilon_1^2}{2g}\right]$$

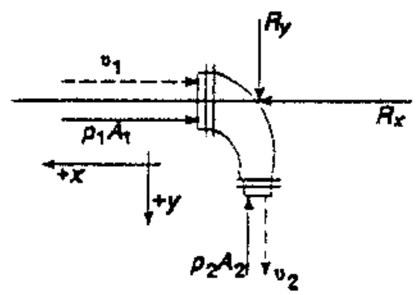
$$p_1 = \frac{62.4\text{ lb}}{\text{ft}^3}\left[\frac{1.12(80)^2 - (20)^2}{64.4}\right]\text{ft} = 6550\text{ lb/ft}^2$$

$$R_x - p_1A_1 = \rho Q(\upsilon_{2_x} - \upsilon_{1_x}) = \rho Q(-\upsilon_2 - (-\upsilon_1)) = \rho Q(\upsilon_1 - \upsilon_2)$$

$$R_x = \rho Q(\upsilon_1 - \upsilon_2) + p_1A_1 = (1.94)(\underbrace{0.0218)(80}_{A\upsilon})(20 - 80) + (6550)(0.0873) = -203 + 571$$

= **368 lb**

16.14 $$\upsilon_1 = \frac{Q}{A_1} = \frac{0.025\text{ m}^3/\text{s}}{7.538\times10^{-3}\text{m}^2} = 3.32\text{ m/s}$$



$$\upsilon_2 = \frac{Q}{A_2} = \frac{0.025\text{ m}^3/\text{s}}{1.945\times10^{-3}\text{ m}^2} = 12.85\text{ m/s}$$

$$\frac{p_1}{\gamma} + \frac{\upsilon_1^2}{2g} - h_L = \frac{p_2}{\gamma} + \frac{\upsilon_2^2}{2g}$$

$$p_2 = p_1 + \gamma\left[\frac{\upsilon_1^2 - \upsilon_2^2}{2g} - h_L\right] = p_1 + \gamma\left[\frac{\upsilon_1^2}{2g} - \frac{\upsilon_2^2}{2g} - 3.5\frac{\upsilon_1^2}{2g}\right] = p_1 + \gamma\left[-\frac{\upsilon_2^2}{2g} - 2.5\frac{\upsilon_1^2}{2g}\right]$$

$$p_2 = 825\text{ kPa} + (1.03)(9.81\text{ kN/m}^3)\left[\frac{[-12.85^2 - 2.5(3.32)^2]\text{m}^2/\text{s}^2}{2(9.81\text{ m/s}^2)}\right]$$

= 825 kPa − 99.2 kPa = 726 kPa

$\rho Q\upsilon_1 = (1.03)(1000\text{ kg/m}^3)(0.025\text{ m}^3)(3.32\text{ m/s}) = 85.5\text{ kg·m/s}^2 = 85.5\text{ N}$

$\rho Q\upsilon_2 = (1.03)(1000)(0.025)(12.85)\text{N} = 331\text{ N}$

$p_1A_1 = (825\ \text{kN/m}^2)(7.538 \times 10^{-3}\ \text{m}^2) = 6.219\ \text{kN} = 6219\ \text{N}$
$p_2A_2 = (726\ \text{kN/m}^2)(1.945 \times 10^{-3}\ \text{m}^2) = 1.412\ \text{kN} = 1412\ \text{N}$

x-direction:

$R_x - p_1A_1 = \rho Q\left(\upsilon_{2_x} - \upsilon_{1_x}\right) = \rho Q(0 - (-\upsilon_1)) = \rho Q\upsilon_1$

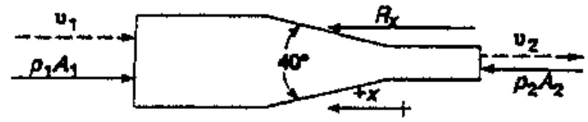
$R_x = \rho Q\upsilon_1 + p_1A_1 = 85.5\ \text{N} + 6219\ \text{N} = \mathbf{6305\ N}$

y-direction:

$R_y - p_2A_2 = \rho Q\left(\upsilon_{2_y} - \upsilon_{1_y}\right) = \rho Q(\upsilon_2 - 0) = \rho Q\upsilon_2$

$R_y = \rho Q\upsilon_2 + p_2A_2 = 331\ \text{N} + 1412\ \text{N} = \mathbf{1743\ N}$

16.15 $Q = 500\ \text{gal/min} \times \dfrac{1\ \text{ft}^3/\text{sec}}{449\ \text{gal/min}}$
$= 1.114\ \text{ft}^3/\text{sec}$



$$\upsilon_1 = \frac{Q}{A_1} = \frac{1.114\ \text{ft}^3/\text{sec}}{0.2006\ \text{ft}^2} = 5.55\ \text{ft/sec} \qquad A_1 = 0.2006\ \text{ft}^2 = 28.89\ \text{in}^2$$

$$A_2 = 0.05132\ \text{ft}^2 = 7.39\ \text{in}^2$$

$$\upsilon_2 = \frac{Q}{A_2} = \frac{1.114\ \text{ft}^3/\text{sec}}{0.05132\ \text{ft}^2} = 21.7\ \text{ft/sec} \qquad D_1/D_2 = 0.5054/0.2557 = 1.98;\ K = 0.043$$

From Section 10.8, $h_L = K\dfrac{\upsilon_2^2}{2g} = 0.043\dfrac{(21.7)^2}{64.4} = 0.314\ \text{ft}$

$$\frac{p_1}{\gamma} + \frac{\upsilon_1^2}{2g} - h_L = \frac{p_2}{\gamma} + \frac{\upsilon_2^2}{2g} \therefore p_2 = p_1 + \gamma\left[\frac{\upsilon_1^2 - \upsilon_2^2}{2g} - 0.314\right]$$

$$p_2 = 125 + \frac{62.4\ \text{lb}}{\text{ft}^3}\left[\frac{(5.55)^2 - (21.7)^2}{64.4} - 0.314\right]\text{ft}\ \frac{1\ \text{ft}^2}{144\ \text{in}^2} = 125 - 3.10 = 121.9\ \text{psig}$$

$\Sigma F_x = R_x + p_2A_2 - p_1A_1 = \rho Q\left(\upsilon_{2_x} - \upsilon_{1_x}\right) = \rho Q(-\upsilon_2 - (-\upsilon_1)) = \rho Q(\upsilon_1 - \upsilon_2)$

$R_x = \rho Q(\upsilon_1 - \upsilon_2) - p_2A_2 + p_1A_1$
$= (1.94)(1.11)(5.55 - 21.7) - (121.9)(7.39) + (125)(28.89)$
$R_x = -34.9 - 900 + 3611 = \mathbf{2676\ lb}$

16.16 Find p_1 ($p_2 = 0$, $\upsilon_1 = \upsilon_2$, $z_1 = z_2$)

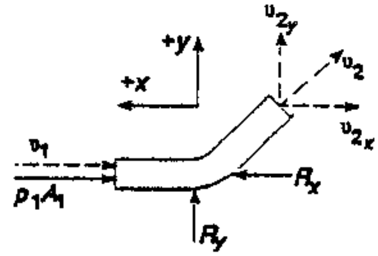
$$\frac{p_1}{\gamma} - h_L = 0;\ p_1 = \gamma h_L = \gamma f_T \frac{L_e}{D}\frac{\upsilon^2}{2g}$$

$\upsilon = Q/A = 6.5/0.3174 = 20.45$ ft/sec

$$N_R = \frac{\upsilon D}{\gamma} = \frac{(20.45)(0.6354)}{9.15\times10^{-6}} = 1.42\times10^6$$

$f_T = 0.014$ (Table 10.5); $L_e/D = 16$ (Table 10.4)

$p_1 = \gamma h_L = (62.2)(0.014)(16)(20.45)^2/64.4 = 90.5$ lb/ft^2



x-direction:

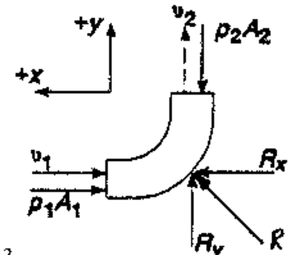
$R_x - p_1A_1 = \rho Q\left(\upsilon_{2_x} - \upsilon_{1_x}\right) = \rho Q(-\upsilon\sin 45° - (-\upsilon_1)) = \rho Q\upsilon(1 - \sin 45°)$

$R_x = \rho Q\upsilon(1 - \sin 45°) + p_1A_1 = (1.93)(6.5)(20.45)(0.293) + (90.5)(0.3174) =$ **103.9 lb**

y-direction:

$R_y = \rho Q\left(\upsilon_{2_y} - \upsilon_{1_y}\right) = \rho Q(\upsilon_2\cos 45° - 0) = (1.93)(6.5)(20.45)(0.707) =$ **182 lb**

16.17 $$\upsilon = \frac{Q}{A} = \frac{0.125\ \text{m}^3/\text{s}}{1.864\times10^{-2}\ \text{m}^2} = 6.71\ \text{m/s}$$



x-direction:

$R_x - p_1A_1 = \rho Q\left(\upsilon_{2_x} - \upsilon_{1_x}\right) = \rho Q(0 - (-\upsilon_1))$

$R_x = \rho Q\upsilon_1 + p_1A_1$

$$= \frac{1000\ \text{kg}}{\text{m}^3}\times\frac{0.125^{\ 3}}{\text{s}}\times\frac{6.71\ \text{m/s}}{\text{s}} + \frac{1050\ \text{kN}}{\text{m}^2}\times 1.864\times10^{-2}\ \text{m}^2$$

$$R_x = 838\ \text{kg·m/s}^2 + 19.57\ \text{kN} = 838\ \text{N}\times\frac{1\ \text{kN}}{10^3\ \text{N}} + 19.57\ \text{kN} = \mathbf{20.41\ kN}$$

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y-direction:

$R_y - p_2A_2 = \rho Q\left(\upsilon_{2_y} - \upsilon_{1_y}\right) = \rho Q(\upsilon_2 - 0)$

$R_y = \rho Q\upsilon_2 + p_2A_2 = 0.838$ kN + 19.57 kN = **20.41 kN**

$R = \sqrt{R_x^2 + r_\upsilon^2}$

$R = 28.9$ kN @ 45°

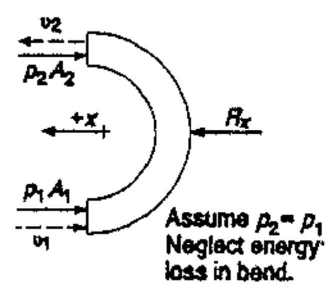
16.18 $Q = 2000$ L/min × 16.67×10^{-6} m^3/s/1 L/min = 0.0333 m^3/s

$\upsilon = Q/A = 0.0333$ m^3/s/7.419×10^{-3} m^2 = 4.49 m/s

$\Sigma F_x = \rho Q\left(\upsilon_{2_x} - \upsilon_{1_x}\right) = \rho Q(\upsilon_2 - (-\upsilon_1)) = 2\rho Q\upsilon$

$\Sigma F_x = R_x - p_1A_1 - p_2A_2 = R_x - 2p_1A_1$

Then $R_x - 2p_1A_1 = 2\rho Q\upsilon$



$R_x = 2\rho Q\upsilon + 2p_1A_1$

$$= 2(0.89)\frac{(1000\ \text{kg})}{\text{m}^3}\times\frac{0.0333\ \text{m}^3}{\text{s}}\times\frac{4.49\ \text{m}}{\text{s}} + 2\times\frac{2.0\times10^6\ \text{N}}{\text{m}^2}\times 7.419\times10^{-3}\text{m}^2$$

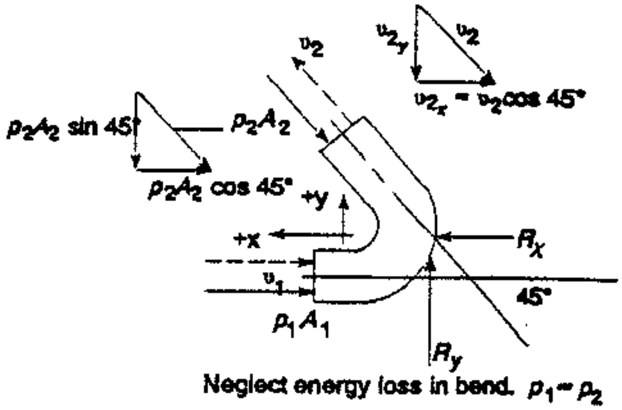
$$= \frac{267\ \text{kg}\cdot\text{m}}{\text{s}^2} + 29.68\times10^3\ \text{N} = 267\ \text{N} + 29.68\times10^3\ \text{N} = 0.267\ \text{kN} + 29.68\ \text{kN}$$

$R_x =$ **29.95 kN**

16.19 $\upsilon_1 = \upsilon_2 = \upsilon$

$$\upsilon = \frac{Q}{A} = \frac{0.12\ \text{m}^3/\text{s}}{1.670 \times 10^{-2}\ \text{m}^2} = 7.19\ \text{m/s}$$

$$p_1A_1 = p_2A_2 = \frac{275\ \text{kN}}{\text{m}^2} \times 1.670 \times 10^{-2}\ \text{m}^2 = 4.59\ \text{kN}$$



x-direction:

$R_x - p_1A_1 - p_2A_2 \cos 45° = \rho Q\left(\upsilon_{2_x} - \upsilon_{1_x}\right)$

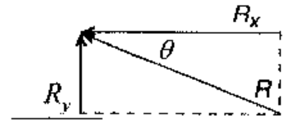
$R_x - p_1A_1(1 + \cos 45°) = \rho Q(\upsilon_2 \cos 45° - (-\upsilon_1))$

$R_x - p_1A_1(1 + \cos 45°) = \rho Q\upsilon(1 + \cos 45°)$

$R_x = \rho Q\upsilon(1 + \cos 45°) + p_1A_1(1 + \cos 45°)$

$$\rho Q\upsilon = \frac{1590\ \text{kg}}{\text{m}^3} \times \frac{0.12\ \text{m}^3}{\text{s}} \times \frac{7.19\ \text{m}}{\text{s}} = \frac{1.37 \times 10^3\ \text{kg} \cdot \text{m}}{\text{s}^2} = 1.37\ \text{kN}$$

$R_x = 1.37\ \text{kN}(1.707) + 4.59\ \text{kN}(1.707) = \mathbf{10.17\ kN}$



$R = \sqrt{R_x^2 + R_y^2} = 11.0\ \text{kN}$

$\theta = \tan^{-1} \dfrac{R_y}{R_x} = 22.3°$

y-direction:

$R_y - p_2A_2 \sin 45° = \rho Q\left(\upsilon_{2_y} - \upsilon_{1_y}\right) = \rho Q(\upsilon_2 \sin 45° - 0) = \rho Q\upsilon \sin 45°$

$R_y = (\rho Q\upsilon + p_2A_2) \sin 45° = (1.37\ \text{kN} + 4.59\ \text{kN})(0.707) = \mathbf{4.18\ kN}$

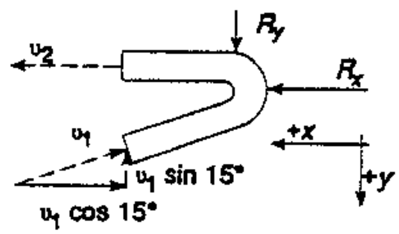
16.20 $Q = A\upsilon = (3.142 \times 10^{-2}\ \text{m}^2)(30\ \text{m/s}) = 0.943\ \text{m}^3/\text{s}$

a. **x-direction:**

$R_x = \rho Q\left(\upsilon_{2_x} - \upsilon_{1_x}\right) = \rho Q(\upsilon_2 - (-\upsilon_1 \cos 15°))$

$R_x = \rho Q\upsilon(1 + \cos 15°) = 1.966\rho Q\upsilon$

$$R_x = (1.966)\left(\frac{1000\ \text{kg}}{\text{m}^3}\right)\left(\frac{0.943\ \text{m}^3}{\text{s}}\right)\left(\frac{30\ \text{m}}{\text{s}}\right)$$



$R_x = 55.6 \times 10^3\ \text{kg·m/s}^2 = 55.6\ \text{kN}$ = Force of car on water. ←

Force on car = 55.6 kN →

y-direction:

$R_y = \rho Q\left(\upsilon_{2_y} - \upsilon_{1_y}\right) = \rho Q(0 - (-\upsilon_1 \sin 15°)) = \rho Q\upsilon \sin 15°$

$R_y = (1000)(0.943)(30)(0.259) = 7.32 \times 10^3\ \text{kg·m/s}^2 = 7.32\ \text{kN}$ = Force on water ↓

Force on car = 7.32 kN ↑

b. Because the inlet jet acts at an angle to the x-y directions, we compute its components: $v_{1x} = v_1 \cos(15°) = (30\text{ m/s})(0.966) = 28.98\text{ m/s}$: $v_{1y} = v_1 \sin(15°) = 30\text{ m/s})(0.259) = 7.76\text{ m/s}$

Only v_{1x} is affected by the moving vane. Then $v_{e/x} = v_{1x} - 12\text{ m/s} = 16.98\text{ m/s}$. $v_{e1y} = v_{1y} = 7.76\text{ m/s}$. The magnitude of the resultant effective velocity is:

$|v_{e1y}| = \sqrt{(16.98)^2 + (7.76)^2} = 18.67\text{ m/s}$

The total effective mass flow rate into the vane, M_e, is,

$M_e = \rho Q_e = \rho A v_{e1} = (1000\text{ kg/m}^3)(3.142 \times 10^{-2}\text{ m}^2)(18.67\text{ m/s}) = 586.6\text{ kg/s}$

The velocity, v_{e1}, acts at an angle α, with respect to the horizontal, where $\alpha = \text{Tan}^{-1}(7.76)/16.98) = 24.58°$

Only the component of v_{e1} acting parallel to the vane is maintained as the jet travels around the vane.

This component is computed using β, the difference between α and the angle of the vane inlet.

$\beta = 24.58° - 15° = 9.58°$

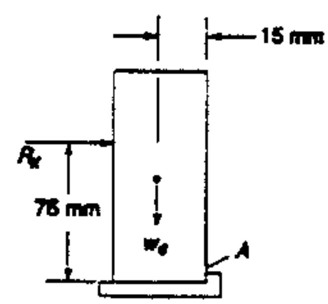
Then, $v_{e1(par)} = (v_{e1})\cos(9.58°) = (18.67\text{ m/s})(0.986) = 18.41\text{ m/s}$

This velocity remains undiminished as the jet travels around the vane. Then $v_{e2} =$ 18.41 m/s to the left.

Force in x-direction: $R_x = M_e(\Delta v_{ex}) = M_e(v_{e2x} - v_{e1x}) = (586.6\text{ kg/s})[18.41 - (-16.98)]\text{m/s}$ $=$ **20.86 kN**

Force in y-direction: $R_y = M_e(\Delta v_{ey}) = M_e(v_{e2y} - v_{e1y}) = (586.6\text{ kg/s})[0 - (-7.76)]\text{m/s}$ $=$ **4.55 kN**

16.21 $R_x = \rho Q\left(\upsilon_{2_x} - \upsilon_{1_x}\right) = \rho A \upsilon_1(0 - (-\upsilon_1)) = \rho A \upsilon_1^2$ ①



Weight of carton $= w_c = \text{mg} = \dfrac{0.10\text{ kg} \cdot 9.81\text{ m}}{\text{s}^2} = 0.981\text{ N}$

$\Sigma M_A = R_x(75\text{ mm}) - w_c(15\text{ mm}) = 0$ Impending tipping

$R_x = w_c \dfrac{15}{75} = 0.981\,N\dfrac{15}{75} = 0.196\text{ N}$

From ①, $\upsilon_1 = \sqrt{\dfrac{R_x}{\rho A}}$

$A = \pi(0.010\text{ m})^2/4 = 7.85 \times 10^{-5}\text{ m}^2$

$$\upsilon_1 = \sqrt{\frac{0.196\text{ kg}\cdot\text{m/s}^2}{(1.20\text{ kg/m}^3)(7.85\times10^{-5}\text{ m}^2)}} = \mathbf{45.6\text{ m/s}}$$

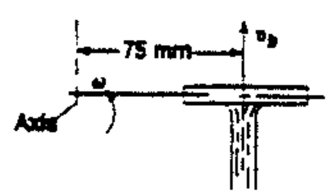
16.22 $R_x = \rho Q\left(\upsilon_{2_x} - \upsilon_{1_x}\right) = \rho A \upsilon_1(0 - (-\upsilon_1)) = \rho A \upsilon_1^2$

$$R_x = \frac{1.20\text{ kg}}{\text{m}^3} \times \frac{\pi(0.015\text{ m})^2}{4} \times \frac{(0.35\text{ m/s})^2}{1} = \frac{2.60\times10^{-5}\text{ kg}\cdot\text{m}}{\text{s}^2} = \mathbf{2.6 \times 10^{-5}\text{ N}}$$

16.23 Let $\upsilon_{1e} = \upsilon_1 - \upsilon_b$

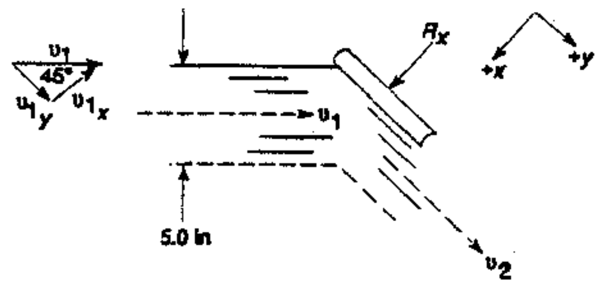
| └ Velocity of Blade

└ Velocity of Air



$$\upsilon_b = R\omega = (0.075\ \text{m}) \times \frac{40\ \text{rev}}{\text{min}} \times \frac{2\pi\ \text{rad}}{\text{rev}} \times \frac{1\ \text{min}}{60\ \text{s}}$$
$$= 0.314\ \text{m/s}$$
$$\upsilon_e = 0.35\ \text{m/s} - 0.314\ \text{m/s} = 0.0358\ \text{m/s}$$
$$R_x = \rho A\, \upsilon_e^2 = (1.20)\frac{\pi(0.015)^2}{4}(0.0358)^2 = \mathbf{2.72 \times 10^{-7}\ N}$$

16.24 $R_x = \rho Q\left(\upsilon_{2_x} - \upsilon_{1_x}\right)$

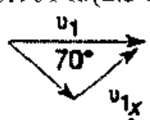


$R_x = \rho Q(0 - (-\upsilon_1 \sin 45°))$
$R_x = \rho Q \upsilon_1 \sin 45°$
$= \rho(A\upsilon_1)\upsilon_1 \sin 45°$
$R_x = \rho A \upsilon_1^2 \sin 45°$
Compute R_x for a 20.0 in length of louver, as shown in Fig. 16.19.
$A = (5\ \text{in})(20.0\ \text{in})(1\ \text{ft}^2/144\ \text{in}^2)$
$= 0.694\ \text{ft}^2$

$$R_x = \frac{2.06 \times 10^{-3}\ \text{lb}\cdot\text{s}^2}{\text{ft}^4} \times 0.694\ \text{ft}^2 \times (10.0\ \text{ft/s})^2 \times \sin 45° = 0.101\ \text{lb}$$

Assume R_x acts at middle of louver, 2.50 in from pivot
Moment = $R_x(2.5)$ = 0.101 lb(2.5 in) = **0.253 lb-in**

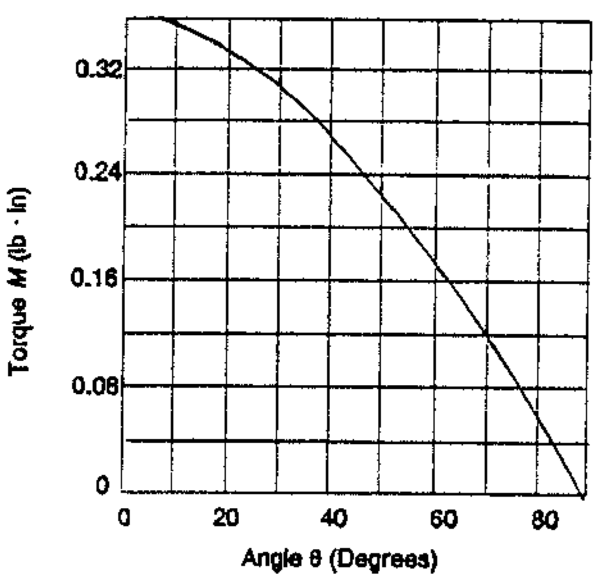
16.25 See Problem 16.24:



$R_x = \rho A\, \upsilon_1^2 \sin 70° = (2.06 \times 10^{-3})(0.694)(10.0)^2 \sin 70° = 0.1345$ lb
Moment = $R_x(2.5)$ = (0.1345 lb)(2.5 in) = **0.336 lb-in**

16.26 See analysis—Problem 16.24: $R_x = \rho A\, \upsilon_1^2 \sin(90 - \theta) = \rho A\, \upsilon_1^2 \cos\theta$
$R_x = (2.06 \times 10^{-3})(0.417)(10.0)^2 \cos\theta = 0.143 \cos\theta$; Moment = $R_x(2.5$ in)

θ	R_x(lb)	M(lb·in)
10	0.141	0.352
20	0.134	0.336
30	0.124	0.310
40	0.110	0.274
50	0.0920	0.230
60	0.0715	0.179
70	0.0489	0.122
80	0.0248	0.062
90	0.0	0.0



16.27 Maximum $F_f = \mu W = 0.60W$

Sliding is impending when $R_x = F_f$

$R_x = \rho Q(\upsilon_{2_x} - \upsilon_{1_x}) = \rho A\upsilon_1(0 - (-\upsilon_1)) = \rho A\,\upsilon_1^2$

Then $W = \dfrac{R_x}{0.6} = \dfrac{\rho A \upsilon_1^2}{0.6}$

$$W = \frac{2.40\times 10^{-3}\ \text{lb}\cdot\text{s}^2}{\text{ft}^4}\times\frac{\pi(1.5\ \text{in})^2}{4}\times\frac{\text{ft}^2}{144\,\text{in}^2}\times\frac{(25.0\ \text{ft/s})^2}{0.6} = 0.0307\ \text{lb}$$

16.28 See Prob. 16.27:

$$W = \frac{\rho A\upsilon_1^2}{0.6} = \frac{1.94\ \text{lb}\cdot\text{s}^2}{\text{ft}^4}\times\frac{\pi(0.75\ \text{in})^2}{4}\times\frac{(25.0\ \text{ft/s})^2}{144\ \text{in}^2/\text{ft}^2}\times\frac{1}{0.6} = 6.20\ \text{lb}$$

16.29 $R_x = M(\Delta v_2) = M(v_{2x} - v_{1x})$: Where $M = \rho A v$

$A = \pi D^2/4 = \pi(0.0075\ \text{m})^2/4 = 4.418\times 10^{-5}\ \text{m}^2$

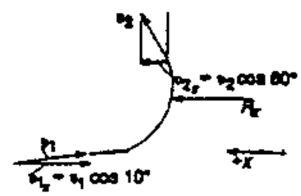
$M = \rho A v = (1000\ \text{kg/m}^3)(4.418\times 10^{-5}\ \text{m}^2)(25\ \text{m/s}) = 1.105\ \text{kg/s}$

$v_{1x} = v_1\cos(10°) = (25\ \text{m/s})(0.985) = 24.62\ \text{m/s}$;

$v_{1y} = v_1\sin(10°) = (25\ \text{m/s})(0.174) = 4.34\ \text{m/s}$

$v_2 = v = 25\ \text{m/s}$;

$v_{2x} = v_2\cos(60°) = (25\ \text{m/s})(0.5) = 12.5\ \text{m/s}$; $v_{2y} = v_2\sin(60°)$
$= (25\ \text{m/s})(0.0866) = 21.65\ \text{m/s}$



Force in the x-direction: $R_x = M(\Delta v_x) = M(v_{2x} - v_{1x})$
$= (1.105\ \text{kg/s})[12.5 - (-24.62)]\text{m/s} = \mathbf{41.0\ N}$

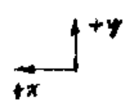
Force in the y-direction: $R_y = M(\Delta v_y) = M(v_{2y} - v_{1y})$
$= (1.105\ \text{kg/s})[21.65 - 4.34]\text{m/s} = \mathbf{19.1\ N}$

16.30 Compute the force on one blade when the turbine wheel is rotating and has a tangential velocity of 10 m/s.

Method: See vector diagram on next page. Law of sines and law of cosines used.

1. Compute the velocity relative to the blade v_{R1} for the inlet.
2. The magnitude of this velocity remains undiminished as the jet traverses the blade.
3. The relative velocity rotates 110° as it traverses the blade.
4. Resolve v_{R1} and v_{R2} into x and y components.
5. Compute the effective mass flow rate $M_e = \rho Q_e = \rho A_j v_R$.
6. Compute reaction forces: $R_x = M_e(v_{R2x} - v_{R1x})$ and $R_y = M_e(v_{R2y} - v_{R1y})$

Results:



$v_{R1} = 15.25$ m/s **16.54°** ; $v_{R2} = 15.25$ m/s **53.46°**

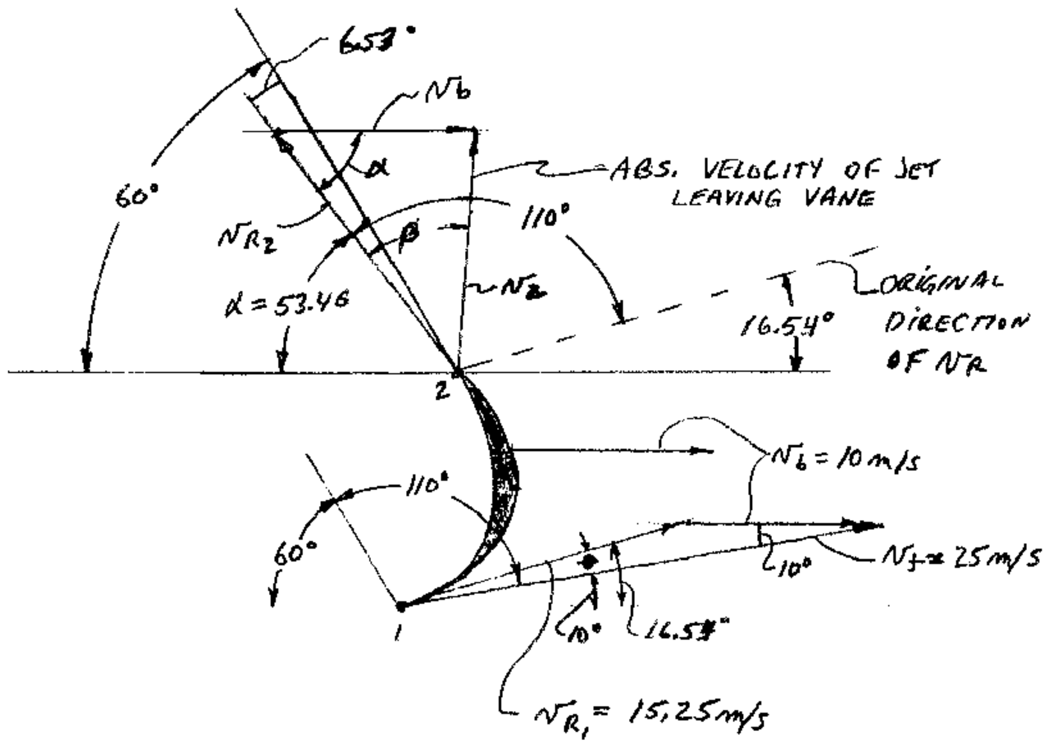
$v_{R1x} = 14.62$ m/s →; $v_{R1y} = 4.34$ m/s ↑; $V_{R2x} = 9.077$ m/s ←; $v_{R2y} = 12.25$ m/s^

$M_e = \rho Q_e = \rho A_j v_R = (1000\ \text{kg/m}^3)[\pi(0.0075\ \text{m})^2/4](15.25\ \text{m/s}) = 0.674\ \text{kg/s}$

$R_x = M_e(V_{R2x} - V_{R1x}) = (0.674\ \text{kg/s})(9.077 - (-14.62))\ \text{m/s} = 15.97\ \text{kg m/s}^2 = 15.97\ \text{N}$ ←

$R_y = M_e(V_{R2y} - V_{R1y}) = (0.674)(12.25 - 4.34) = 5.33$ N↑

$$\text{Rotational speed} = \omega = \frac{\upsilon_t}{r} = \frac{10.0\ \text{m}}{\text{s}(0.20\ \text{m})}\times\frac{\text{rev}}{2\pi\ \text{rad}}\times\frac{60\ \text{s}}{\text{min}} = 477\ \text{rpm}$$



Vector diagram for Problem 16.30

V_j = Velocity of water jet

v_R = Velocity relative to blade

v_b = Velocity of blade

Assume $|v_{R1}| = |v_{R2}|$ and v_R vector is rotated 110° from inlet to outlet

Problem 16.31			Forces on rotating turbine wheel				
v Blade (m/s)	v_{R1} (m/s)	ϕ (°)	v_{R2} (m/s)	α (°)	M_e (kg/s)	R_x (N)	R_y (N)
0	25.00	10.00	25.00	60.00	1.105	41.00	19.12
5	20.09	12.48	20.09	57.52	0.888	27.00	11.20
10	15.25	16.54	15.25	53.46	0.674	15.97	5.33
15	10.55	24.29	10.55	45.71	0.466	7.92	1.50
20	6.34	43.22	6.34	26.78	0.280	2.88	−0.42
25	4.36	95.00	4.36	−25.00	0.193	0.69	−1.19

R_x is the reaction force exerted by the blade on the water; positive R_x acts to the left
Then the force exerted by the water on the blade acts to the right, accelerating the blade
Positive R_y acts radially outward
Then the force exerted by the water on the blade acts radially inward toward the center of rotation
When R_y is negative, the net radial force on the blade is outward.