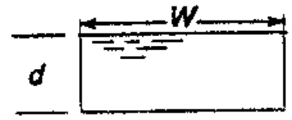
CHAPTER FOURTEEN

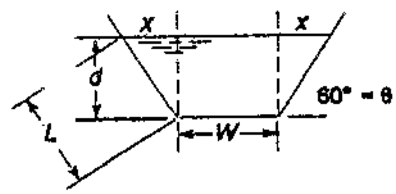
OPEN CHANNEL FLOW

14.1 $R = \dfrac{A}{WP} = \dfrac{\pi D^2}{8} \times \dfrac{2}{\pi D} = \dfrac{D}{4} = \dfrac{300\text{ mm}}{4}$ = **75 mm**

14.2 $R = \dfrac{A}{WP} = \dfrac{Wd}{W + 2d} = \dfrac{(2.75\text{ m})(0.05\text{ m})}{[2.75 + 2(0.50)]\text{m}}$ = **0.367 m**



14.3 $d = 1.50$ ft; $W = 3.50$ ft, $x = d \tan 30° = 0.866$ ft
$L = d/\cos 30° = 1.732$ ft



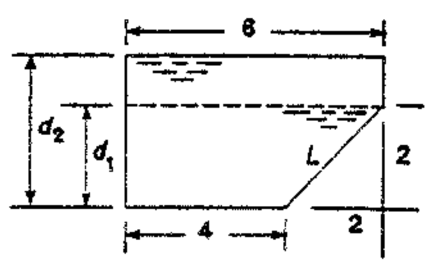
$$A = Wd + 2\left[\frac{1}{2}xd\right] = (3.50)(1.50) + (\underset{x}{0.866})(\underset{d}{1.50}) = 6.549\text{ ft}^2$$

$WP = W + 2L = 3.50 + 2(1.732) = 6.964$ ft
$R = A/WP = 6.549\text{ ft}^2/6.964\text{ ft}$ = **0.940 ft**

14.4 Data from Prob. 14.3, but $\theta = 45°$. $x = d \tan 45° = d = 1.50$ ft
$L = d/\cos 45° = 1.50/\cos 45° = 2.121$ ft
$A = (3.50)(1.50) + (1.50)(1.50) = 7.50\text{ ft}^2$
$WP = 3.50 + 2(2.121) = 7.743$ ft
$R = A/WP = 7.50\text{ ft}^2/7.743\text{ ft}$ = **0.969 ft**

14.5 $W = 150$ mm; $d = 62$ mm; $X = 1.5d = 1.5(62) = 93$ mm
$L = \sqrt{X^2 + d^2} = 111.8\text{ mm}^2$
$R = \dfrac{A}{WP} = \dfrac{Wd + Xd}{W + 2L} = \dfrac{(150)(62) + (93)(62)}{150 + 2(111.8)} = \dfrac{15066\text{ mm}^2}{373.5\text{ mm}}$ = **40.3 mm**

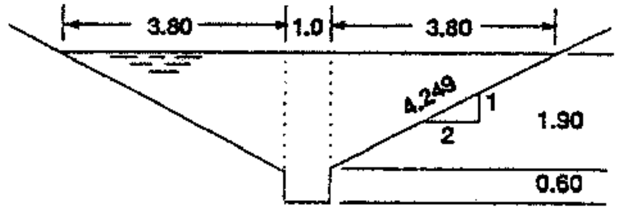
14.6 $d = d_1 = 2.0$ in; $L = \sqrt{2^2 + 2^2} = 2.828$ in
$A = (4)(2) + \dfrac{1}{2}(2)(2) = 10\text{ in}^2$
$WP = 4 + 2 + 2.828 = 8.828$ in
$R = A/WP$ = **1.133 in**



14.7 Data from Prob. 14.6. $d = d_2 = 3.50$ in
$A = (4)(2) + \dfrac{1}{2}(2)(2) + (6)(3.50 - 2.00) = 19.0\text{ in}^2$
$WP = 3.50 + 4 + 2.828 + 1.50 = 11.828$ in
$R = A/WP = 19.0\text{ in}^2/11.828\text{ in}$ = **1.606 in**

14.8 $A = (1.0)(0.5) = 0.50\text{ m}^2$; $WP = 1.0 + 2(0.5) + 2.0$ m
$R = A/WP = 0.50/2.0 = \mathbf{0.25\ m}$

14.9 $A = (1.0)(2.50) + 2\left[\frac{1}{2}(1.9)(3.8)\right]$
$= 9.72\text{ m}^2$
$WP = 2(4.249) + 2(0.60) + 1.0$
$= 10.697$ m
$R = A/WP = \mathbf{0.909\ m}$

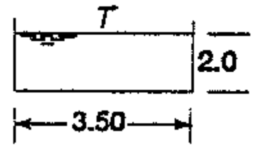


14.10 $A = (3.50)(2.0) = 7.00\text{ m}^2$; $WP = 3.50 + 2(2.0) = 7.50$ m
$R = A/WP = 0.933$ m; $n = 0.017$

$$\upsilon = \frac{1.00}{n}R^{2/3}S^{1/2} = \frac{1.00}{0.017}(0.933)^{2/3}(0.001)^{1/2} = 1.777\text{ m/s}$$

$Q = A\upsilon = (7.00\text{ m}^2)(1.777\text{ m/s}) = \mathbf{12.44\ m^3/s}$
$y_h = A/T = 7.00\text{ m}^2/3.50\text{ m} = 2.00$ m

$$N_F = \upsilon/\sqrt{gy_h} = \frac{1.777\text{ m/s}}{\sqrt{(9.81\text{ m/s}^2)(2.00\text{ m})}} = \mathbf{0.401}$$



14.11 See Prob. 14.7. $d_2 = 3.50$ in; $A = 19.0\text{ in}^2$; $WP = 11.828$ in; $R = 1.606$ in
$A = 19.0\text{ in}^2(1\text{ ft}^2/144\text{ in}^2) = 0.132\text{ ft}^2$

$$WP = 11.828\text{ in} \times \frac{1\text{ ft}}{12\text{ in}} = 0.986\text{ ft}$$

$$R = 1.606\text{ in} \times \frac{1\text{ ft}}{12\text{ in}} = 0.134\text{ ft}$$

$$S = \frac{h}{L} = \frac{4.0\text{ in}}{60\text{ ft}} \times \frac{\text{ft}}{12\text{ in}} = 0.00556;\ n = 0.013\text{ given}$$

$$Q = \frac{1.49}{n}AR^{2/3}S^{1/2} = \frac{1.49}{0.013}(0.132)(0.134)^{2/3}(0.00556)^{1/2} = \mathbf{0.295\ ft^3/s}$$

14.12 $S = 1\text{ ft}/500\text{ ft} = 0.002$; $n = 0.024$; $A = \frac{\pi D^2}{8} = \frac{\pi(6)^2}{8} = 14.14\text{ ft}^2$
$WP = \pi D/2 = \pi(6)/2 = 9.425$ ft; $R = A/WP = 1.50$ ft

$$Q = \frac{1.49}{n}AR^{2/3}S^{1/2} = \frac{1.49}{0.024}(14.14)(1.50)^{2/3}(0.002)^{1/2} = \mathbf{51.4\ ft^3/s}$$

14.13 $A = (0.205)(0.250) = 0.05125\text{ m}^2$; $WP = 0.205 + 2(0.250) = 0.705$ m
$R = A/WP = 0.0727$ m; $n = 0.012$; $Q = \frac{1.00}{n}1\ AR^{2/3}S^{1/2}$

$$S = \left[\frac{nQ}{AR^{2/3}}\right]^2 = \left[\frac{(0.012)(0.0833)}{(0.05125)(0.0727)^{2/3}}\right]^2 = \mathbf{0.0125}$$

where $Q = 5000\text{ L/min} \times \frac{1\text{ m}^3/\text{s}}{60000\text{ L/min}} = \mathbf{0.0833\ m^3/s}$

14.14 See Prob. 14.8 for $d = 0.50$ m; Prob. 14.9 for $d = 2.50$ m

$S = 0.50\% = 0.005$, $n = 0.017$

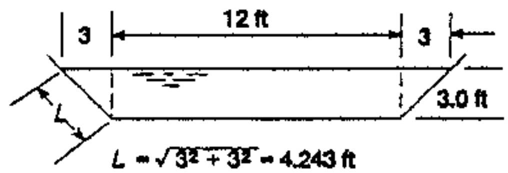
a. $d = 0.50$ m; $A = 0.50$ m^2; $R = 0.25$ m

$$Q = \frac{1.00}{n} AR^{2/3}S^{1/2} = \frac{1.00}{0.017}(0.50)(0.25)^{2/3}(0.005)^{1/2} = \mathbf{0.825\ m^3/s}$$

b. $d = 2.50$ m; $A = 9.72$ m^2; $R = 0.909$ m

$$Q = \frac{1.00}{0.017}(9.72)(0.909)^{2/3}(0.005)^{1/2} = \mathbf{37.9\ m^3/s}$$

14.15 a. **Depth = 3.0 ft:**



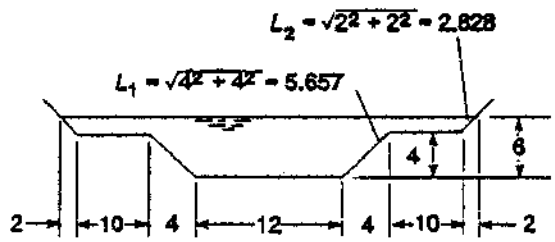
$$A = (3)(12) + 2\left[\frac{1}{2}(3)(3)\right] = 45\ \text{ft}^2$$

$WP = 12 + 2(4.243) = 20.485$ ft

$R = A/WP = 45/20.485 = 2.197$ ft

$$Q = \frac{1.49}{0.04}(45)(2.197)^{2/3}(0.00015)^{1/2} = \mathbf{34.7\ ft^3/s}$$

b. **Depth = 6.0 ft:**



$$A = (4)(12) + 2\left[\frac{1}{2}(4)(4)\right] + (2)(40) + 2\left[\frac{1}{2}(2)(2)\right]$$

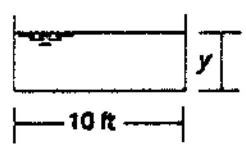
$A = 148$ ft^2

$WP = 2(2.828) + 2(10) + 2(5.657) + 12 = 48.97$ ft

$R = A/WP = 148/48.97 = 3.022$ ft

$$Q = \frac{1.49}{0.04}(148)(3.022)^{2/3}(0.00015)^{1/2} = \mathbf{141.1\ ft^3/s}$$

14.16 $$AR^{2/3} = \frac{nQ}{1.49S^{1/2}} = \frac{(0.015)(150)}{(1.49)(0.001)^{1/2}} = 47.75$$



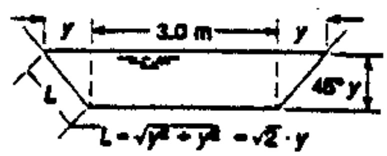
$$A = 10y;\ WP = 10 + 2y;\ R = \frac{A}{WP} = \frac{10y}{10 + 2y}$$

$$AR^{2/3} = 10y\left[\frac{10y}{10 + 2y}\right]^{2/3}$$ Find y such that $AR^{2/3} = 47.75$

By trial and error, $\mathbf{y = 3.10\ ft}$;

$$AR^{2/3} = 10(3.1)\left[\frac{10(3.1)}{10 + 2(3.1)}\right]^{2/3} = 47.78$$

14.17 $AR^{2/3} = \dfrac{nQ}{1.00S^{1/2}} = \dfrac{(0.017)(15)}{(0.001)^{1/2}} = 8.064$



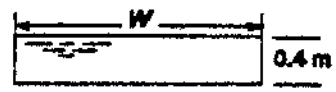
$A = 3.0(y) + 2\left[\dfrac{1}{2}(y)(y)\right] = 3y + y^2$

$WP = 3.0 + 2L = 3.0 + 2\sqrt{2}y$

$AR^{2/3} = (3y + y^2)\left[\dfrac{3y + y^2}{3 + 2.828y}\right]^{2/3}$: By trial,

for y = **1.69 m**, $AR^{2/3} = 8.02$

14.18 $S = 0.075/50 = 0.0015$



$AR^{2/3} = \dfrac{nQ}{1.00S^{1/2}} = \dfrac{(0.013)(2.0)}{(0.0015)^{1/2}} = 0.671$

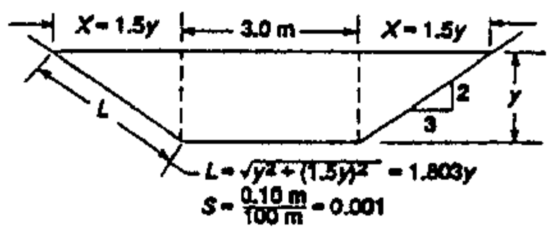
$A = 0.4W$; $WP = W + 0.8$

$R = \dfrac{A}{WP} = \dfrac{0.4W}{W + 0.8}$: Then $AR^{2/3} = 0.4W\left[\dfrac{0.4W}{W + 0.8}\right]^{2/3}$: By trial, W = **3.55 m**

14.19 For $y = 1.50$ m, $X = 2.25$ m

$L = 1.803(1.5) = 2.704$ m

$A = (3)(1.5) + 2\left[\dfrac{1}{2}(1.5)(2.25)\right] = 7.875\ \text{m}^2$



$WP = 3.0 + 2(2.704) = 8.408$ m

$R = A/WP = 7.875/8.408 = 0.937$ m

$Q = \dfrac{1.00}{0.015}(7.875)(0.937)^{2/3}(0.001)^{1/2} = \mathbf{15.89\ m^3/s}$

$v = \dfrac{Q}{A} = \dfrac{15.89\ \text{m}^3/\text{s}}{7.875\ \text{m}^2} = 2.018$ m/s; Top width $= T = 3.0 + 2X = 7.50$ m

Hydraulic Depth $= y_h = \dfrac{A}{T} = \dfrac{7.875\ \text{m}^2}{7.50\ \text{m}} = 1.05$ m

Froude No. $= N_F = \dfrac{v}{\sqrt{gy_h}} = \dfrac{2.018}{\sqrt{(9.81)(1.05)}} = \mathbf{0.629}$

Find critical depth for $Q = 15.89\ \text{m}^3/\text{s}$: Let $N_F = 1.0 = \dfrac{Q/A}{\sqrt{gy_h}}$ (1)

$A = 3y + Xy = 3y + (1.5y)(y) = 3y + 1.5y^2$

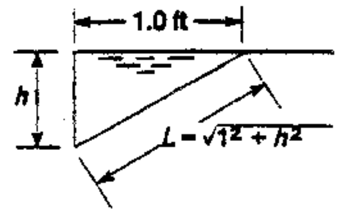
$T = 3 + 2X = 3 + 2(1.5)y = 3 + 3y$

From Eq.(1), $\dfrac{Q}{A} = \dfrac{\sqrt{gA}}{T}$ or $\dfrac{Q^2}{A^2} = \dfrac{gA}{T}$ or $\dfrac{T}{A^3} = \dfrac{g}{Q^2} = \dfrac{9.81}{(15.89)^2} = 0.0388$

But $\dfrac{T}{A^3} = \dfrac{3 + 3y}{\left[3y + 1.5y^2\right]^3}$: Find y such that $\dfrac{T}{A^3} = 0.0388$

By trial, y = **1.16 m** $= y_c$ = Critical depth

14.20 Each trough:



$$Q = 250 \text{ gal/min} \; \frac{1 \text{ ft}^3/\text{s}}{449 \text{ gal/min}} = 0.557 \text{ ft}^3/\text{s}$$

$$AR^{2/3} = \frac{nQ}{1.49S^{1/2}} = \frac{(0.017)(0.557)}{(1.49)(0.01)^{1/2}} = 0.0635$$

$$A = (1.0)(h)\frac{1}{2} = h/2$$

$$WP = h + L = h + \sqrt{1 + h^2}$$

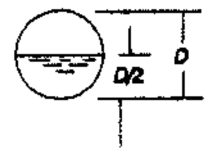
$$R = \frac{A}{WP} = \frac{h/2}{h + \sqrt{1 + h^2}}$$

$$AR^{2/3} = \frac{h}{2}\left[\frac{h/2}{h + \sqrt{1 + h^2}}\right]^{2/3}$$

Find h such that $AR^{2/3} = 0.0635$

By trial, $\boldsymbol{h}$ **= 0.458 ft**

14.21 $Q = 500 \text{ gal/min} \times \dfrac{1 \text{ ft}^3/\text{s}}{449 \text{ gal/min}} = 1.114 \text{ ft}^3/\text{s}$



$$AR^{2/3} = \frac{nQ}{1.49S^{1/2}} = \frac{(0.013)(1.114)}{(1.49)(0.001)^{1/2}} = 0.307$$

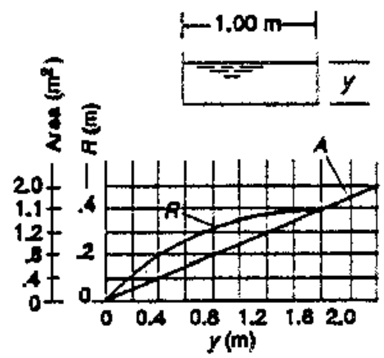
$$A = \pi D^2/8;\; WP = \pi D/2;\; R = \frac{A}{WP} = \frac{\pi D^2}{8}\frac{2}{\pi D} = \frac{D}{4}$$

$$AR^{2/3} = \frac{\pi D^2}{8}\left[\frac{D}{4}\right]^{2/3} = \frac{\pi D^2 (D)^{2/3}}{8(4)^{2/3}} = \frac{\pi (D)^{8/3}}{8(2.52)} = 0.156(D)^{8/3} = 0.307$$

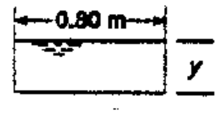
Then $D = \left[\dfrac{0.307}{0.156}\right]^{3/8}$ = **1.29 ft**

14.22 $A = 1.00(y)$; $WP = 1.00 + 2y$; $R = \dfrac{y}{1+2y}$

y(m)	A(m^2)	R(m)	y	A	R
0.10	0.10	0.0833	0.80	0.80	0.3077
0.20	0.20	0.1429	1.00	1.00	0.3333
0.30	0.30	0.1875	1.50	1.50	0.3750
0.40	0.40	0.2222	2.00	2.00	0.4000
0.50	0.50	0.250			
0.60	0.60	0.2727			



14.23 $Q = Av = (0.80\text{ m})(y)(v)$



$$y = \frac{Q}{0.8v} = \frac{2.00\text{ m}^3/\text{s}}{(0.80\text{ m})(3.0\text{ m/s})} = \mathbf{0.833\text{ m}}$$

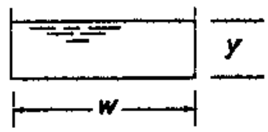
$WP = 0.8 + 2y = 0.8 + 2(0.833) = 2.467\text{ m}$; $A = 0.8(y) = 0.8(0.833) = 0.666\text{ m}^2$

$$R = \frac{A}{WP} = \frac{0.666\text{ m}^2}{2.467\text{ m}} = \mathbf{0.270\text{ m}}$$

14.24 $Q = \frac{1.00}{n} AR^{2/3}S^{1/2}$; $S = \left[\frac{nQ}{AR^{2/3}}\right]^2 = \left[\frac{(0.015)(2.00)}{(0.666)(0.270)^{2/3}}\right]^2 = \mathbf{0.0116\text{ m}^3}$

Data from Problem 14.23.

14.25 and 14.26



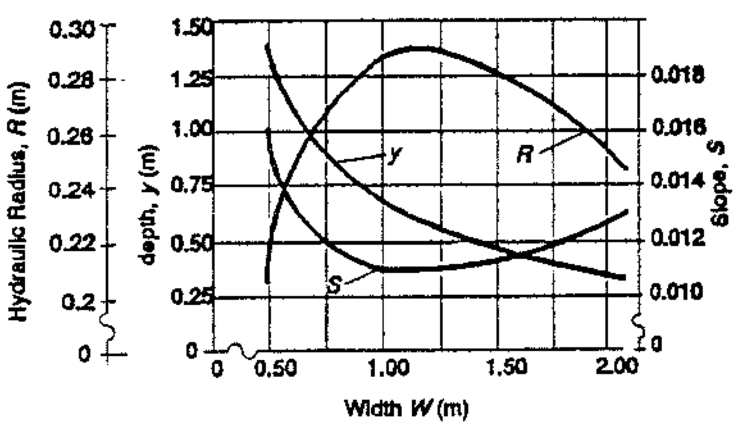
$$Q = Av = Wyv;\ y = \frac{Q}{Wv} = \frac{2.00}{W(3.0)}$$

$$WP = W + 2y;\ R = \frac{A}{WP}$$

$$S = \left[\frac{nQ}{AR^{2/3}}\right]^2 = \left[\frac{(0.015)(2.00)}{AR^{2/3}}\right]^2$$

W(m)	$y = 2/3W$	$WP = W + 2y$	$A = Wy$	$R = A/WP$	S
0.50	1.333 m	3.167 m	0.667 m²	0.2105 m	0.0162
0.75	0.889	2.528	0.667	0.2637	0.0120
1.00	0.667	2.333	0.667	0.2857	0.0108
1.25	0.533	2.317	0.667	0.2878	0.0107
1.50	0.444	2.389	0.667	0.2791	0.0111
1.75	0.381	2.512	0.667	0.2654	0.0119
2.00	0.333	2.667	0.667	0.2500	0.0129

Prob. 14.25 (y and R columns) — Prob. 14.26 (S column)



14.27 From Table 14.2:

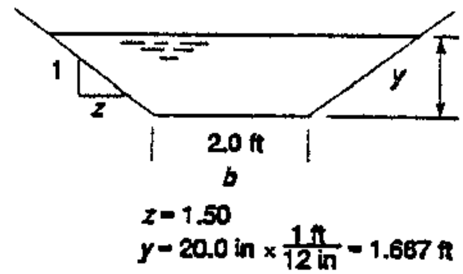
$A = (b + zy)(y) = [2.00 + 1.5(1.667)](1.667)$

$= \mathbf{7.50\ ft^2}$

$WP = b + 2y\sqrt{1 + z^2}$

$= 2.0 + 2(1.667)\sqrt{1 + 1.5^2} = 8.009$ ft

$R = A/WP = 7.50/8.009 = \mathbf{0.936\ ft}$



14.28 **Data from Prob. 14.27** $n = 0.017$ – formed unfinished concrete.

$$Q = \frac{1.49}{n} AR^{2/3}S^{1/2} = \frac{1.49}{0.017}(7.5)(0.936)^{2/3}(0.005)^{1/2} = \mathbf{44.49\ ft^3/s}$$

14.29 Same as 14.28 except $n = 0.010$ – plastic:

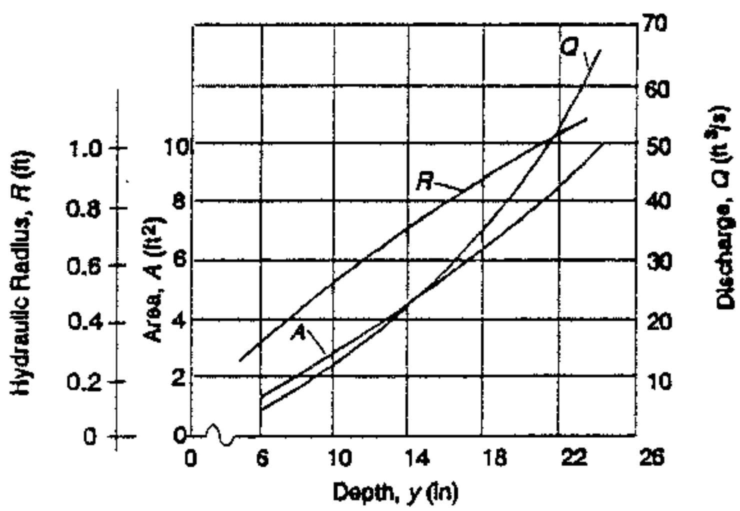
$$Q = \frac{1.49}{0.010}(7.5)(0.936)^{2/3}(0.005)^{1/2} = \mathbf{75.63\ ft^3/s}$$

14.30 and 14.31

$b = 2.00$ ft; $z = 1.50$; $S = 0.005$; $n = 0.017$

$A = (b + zy)y$; $WP = b + 2y\sqrt{1 + z^2}$; $R = A/WP$; $Q = \frac{1.49}{n} AR^{2/3}S^{1/2}$

y(in)	y(ft)	A(ft^2)	WP(ft)	R(ft)	Q(ft^3/s)
6.00	0.500	1.375	3.803	0.3616	4.325
10.00	0.833	2.708	5.005	0.5412	11.147
14.00	1.167	4.375	6.206	0.7049	21.476
18.00	1.500	6.375	7.408	0.8605	35.745
22.00	1.833	8.708	8.610	1.0114	54.380
24.00	2.000	10.000	9.211	1.0856	65.466



14.32 $D = 0.375$ m; $y = 0.225$ m $> D/2 \rightarrow$ See Table 14.2.

$\theta = \pi + 2\sin^{-1}[(2y/D) - 1] = 3.544$ rad

$$A = \frac{(\theta - \sin\theta)D^2}{8} = \frac{(3.544 - \sin 3.544)(0.375)^2\ \text{m}^2}{8} = \mathbf{0.06919\ m^2}$$

$WP = \theta D/2 = (3.544)(0.375\text{ m})/2 = 0.665$ m

$$R = \left[\frac{\theta - \sin\theta}{\theta}\right]\frac{D}{4} = \left[\frac{3.544 - \sin 3.544}{3.544}\right]\frac{0.375\text{ m}}{4} = \mathbf{0.104\ m}$$

14.33 $D = 0.375$ m; $y = .135$ m $< D/2 \rightarrow$ See Table 14.2.

$$\theta = \pi - 2\sin^{-1}[1 - 2y/D] = \pi - 2\sin^{-1}\left[1 - \frac{2(0.135)}{0.375}\right] = 2.574\text{ rad}$$

$$A = \frac{(\theta - \sin\theta)D^2}{8} = \frac{(2.574 - \sin 2.574)(0.375)^2}{8} = \mathbf{0.0358\ m^2}$$

$WP = \theta D/2 = (2.574)(0.375\text{ m})/2 = 0.482$ m

$$R = \left[\frac{\theta - \sin\theta}{\theta}\right]\frac{D}{4} = \left[\frac{2.574 - \sin 2.574}{2.574}\right]\frac{0.375\text{ m}}{4} = \mathbf{0.0742\ m}$$

14.34 $S = 0.0012$; $n = 0.013$; Data from Prob. 14.32:

$$Q = \frac{1.00}{0.013}AR^{2/3}A^{1/2} = \frac{1.00}{0.013}(0.06919)(0.104)^{2/3}(0.0012)^{1/2} = \mathbf{4.08 \times 10^{-2}\ m^3/s}$$

14.35 $S = 0.0012$; $n = 0.013$; Data from Prob. 14.33:

$$Q = \frac{1.00}{0.013}AR^{2/3}S^{1/2} = \frac{1.00}{0.013}(0.0358)(0.0742)^{2/3}(0.0012)^{1/2} = \mathbf{1.68 \times 10^{-2}\ m^3/s}$$

14.36, 14.37, 14.38

$Q = 1.25$ ft^3/s; $\upsilon = 2.75$ ft/s; $A = Q/\upsilon = 0.4545$ ft^2 (All)

Rectangle: $y = \sqrt{\dfrac{A}{2.0}} = \sqrt{\dfrac{0.4545}{2}} = \mathbf{0.4767\ ft}$; $\mathbf{b = 2y = 0.9535\ ft}$

$R = y/2 = 0.4767\text{ ft}/2 = 0.2384$ ft

$$S = \left[\frac{nQ}{1.49\,AR^{2/3}}\right]^2 = \left[\frac{(0.015)(1.25)}{(1.49)(0.4545)R^{2/3}}\right]^2 = \left[\frac{0.02768}{R^{2/3}}\right]^2 = \left[\frac{0.02768}{(0.2384)^{2/3}}\right]^2$$

$= \mathbf{0.00519}$

$$y_h = \frac{A}{T} = \frac{2y^2}{2y} = y = 0.4767\text{ ft: } N_F = \frac{\upsilon}{\sqrt{gy_h}} = \frac{2.75}{\sqrt{(32.2)(0.4767)}}$$

$= \mathbf{0.702} < 1.0$ **Subcritical**

Triangle: $y = \sqrt{A} = \sqrt{0.4545} = \mathbf{0.674\ ft}$; $R = 0.354y = 0.2387$ ft

$$S = \left[\frac{0.02768}{(0.2387)^{2/3}}\right]^2 = \mathbf{0.00518}$$

$$y_h = \frac{A}{T} = \frac{y^2}{2y} = \frac{y}{2} = 0.337\text{ ft: } N_F = \frac{2.75}{\sqrt{(32.2)(0.337)}} = \mathbf{0.835} < 1.0\ \textbf{Subcritical}$$

Trapezoid: $y = \sqrt{\dfrac{A}{1.73}} = \sqrt{\dfrac{0.4545}{1.73}}$ = **0.5126 ft**; $R = y/2 = 0.2563$ ft

$$S = \left[\frac{0.02768}{(0.2563)^{2/3}}\right]^2 = \mathbf{0.00471};$$

$$y_h = \frac{A}{T} = \frac{A}{2.309y} = \frac{0.4545}{2.309(0.5126)} = 0.3841 \text{ ft}$$

$$N_F = \frac{2.75}{\sqrt{(32.2)(0.3841)}} = \mathbf{0.782} < 1.0 \textbf{ Subcritical}$$

Semicircle: $A = \dfrac{\pi y^2}{2}; y = \sqrt{\dfrac{2A}{\pi}} = \sqrt{\dfrac{2(0.4545)}{\pi}}$ = **0.5379 ft**

$$R = y/2 = 0.269 \text{ ft: } S = \left[\frac{0.02768}{(0.269)^{2/3}}\right]^2 = \mathbf{0.00441}$$

$$y_h = \frac{A}{T} = \frac{\pi y^2}{2(2y)} = \frac{\pi y}{4} = 0.4225 \text{ ft}$$

$$N_F = \frac{2.75}{\sqrt{(32.2)(0.4225)}} = 0.746 < 1.0 \textbf{ Subcritical}$$

14.39 a. When $y = y_c$, $N_F = 1.0 = \dfrac{\upsilon}{\sqrt{gy_h}} = \dfrac{Q}{A\sqrt{gy}} = \dfrac{Q}{(by)\sqrt{gy}} = \dfrac{Q}{b\sqrt{g}\, y^{3/2}}$

Then $y = \left[\dfrac{Q}{b\sqrt{g}N_F}\right]^{2/3} = \left[\dfrac{5.5}{2.0\sqrt{9.81}(1.0)}\right]^{2/3}$ = **0.917 m** = y_c

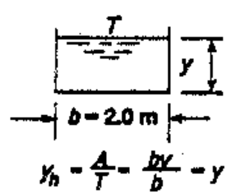
b. Minimum E occurs when $y = y_c$: From Eq. 14.18:

$$E_{min} = y_c + \frac{Q^2}{2gA^2} = y_c + \frac{Q^2}{2g(by_c)^2} = y_c + \frac{Q^2}{2gb^2y_c^2}$$

$$= 0.917 + \frac{5.5^2}{2(9.81)(2.0)^2(.917)^2}$$

E_{min} **= 1.375 m**

c. See spreadsheet and graph for values of y versus E.



d. For $y = 0.50$ m; $E = 0.50 + \dfrac{5.5^2}{2(9.81)(2.0)^2(0.5)^2}$

= **2.042 m**

From spreadsheet, alternate depth = 1.934 m

e. $\upsilon = \dfrac{Q}{A} = \dfrac{Q}{by} = \dfrac{5.5}{2.0y}$; $N_F = \dfrac{\upsilon}{\sqrt{gy}}$

For $y = 0.5$ m, υ = **5.50 m/s**; N_F **= 2.48**

For $y = 1.934$ m, υ = **1.418 m/s**; N_F **= 1.325**

14.39 (continued)

Rectangular channel

$b = 2$		$Q = 5.5$	$n = 0.017$
$\mathbf{E = y + 0.3854/y^2}$		Only for this problem	
y =	E_{min}=		
0.0917	1.375	Critical depth–See Solution Manual	

d), e), f)

Given y =	***E***	***A***	**Velocity**	N_F	***R***	***S***	
0.50	2.04160	1.000	5.500	2.483	0.333	0.0378	Given y
1.9391	2.04160	3.878	1.418	0.325	0.660	0.0010	Alternate depth for given depth by iteration on y to make E the same as for the given y

y	***E***
0.5	154.2
0.1	38.6
0.20	9.835
0.40	2.809
0.60	1.671
0.80	1.402
1.00	1.385
1.20	1.468
1.40	1.597
1.60	1.751
1.80	1.919
2.00	2.096
2.20	2.280
2.40	2.467
2.60	2.657
2.80	2.849
3.00	3.043
3.20	3.238
3.40	3.433
3.60	3.630
3.80	3.827
4.00	4.024
4.20	4.222
4.40	4.420
4.60	4.618

c) Equation for E (in m) as a function of y: $A = by$; $A^2 = b^2y^2$

Given: $b = 2.0$ m; $Q = 5.5$ m^3/s; $v = Q/A$

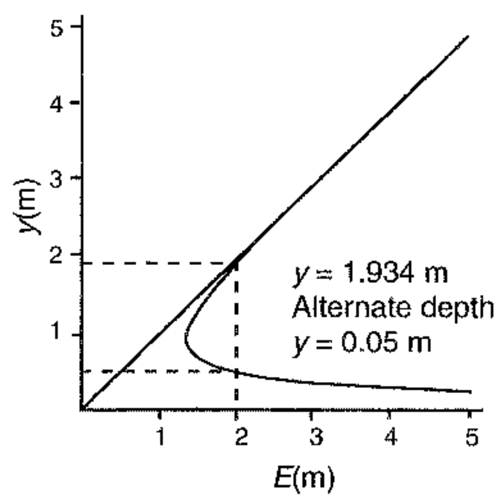
$$E = y + v^2/2g = y + Q^2/2gA^2 = Q^2/2gb^2y^2$$

$$E = y = (5.5)^2 / [3(9.81)(2.0)^2y^2] = y + 0.3854/y^2$$

f) $$S = \left[\frac{nQ}{AR^{2/3}}\right]^2; A = by = 2.0y$$

$$WP = b + 2y = 2 + 2y$$

$$R = \frac{A}{WP} = \frac{2.0y}{2+2y} = \frac{y}{1+y}$$



14.40 **Circular Channel**

$Q =$ 1.45 m^3/s $\quad n =$ 0.015 Finished concrete

$D =$ 1.20 m

$y(m)$	θ(rad)	A(m^2)	T(m)	y_h(m)	N_F	$E(m)$	Velocity (m/s)	
y* less than *D								
0.10	1.171	0.0450	0.6633	0.068	39.478	52.982	32.211	
0.20	1.682	0.1239	0.8944	0.139	10.039	7.181	11.703	
0.25	1.896	0.1707	0.9747	0.175	6.481	3.928	8.494	
0.30	2.094	0.2211	1.0392	0.213	4.539	2.492	6.558	
0.40	2.462	0.3300	1.1314	0.292	2.597	1.384	4.394	
0.50	**2.807**	**0.4460**	**1.1832**	**0.377**	**1.690**	**1.039**	**3.251**	**Given *y***
0.60	3.142	0.5655	1.2000	0.471	1.193	0.935	2.564	
y* greater than *D								
0.658	3.335	0.6349	1.1944	0.532	1.000	0.924	2.284	**Critical depth**
0.70	3.476	0.6849	1.1832	0.579	0.888	0.928	2.117	
0.80	3.821	0.8010	1.1314	0.708	0.687	0.967	1.810	
0.90	4.189	0.9099	1.0392	0.876	0.544	1.029	1.594	
0.913	**4.238**	**0.9231**	**1.0240**	**0.901**	**0.528**	**1.039**	**1.571**	**Alt depth for given *y***
1.00	4.601	1.0071	0.8944	1.126	0.433	1.106	1.440	
1.199	6.168	1.1309	0.0693	16.330	0.101	1.283	1.282	**Nearly full pipe depth**
1.20	6.283	1.1310	0.0000	#DIV/0!	#DIV/0!	1.284	1.282	**Full pipe** (y_h and N_F undefined)

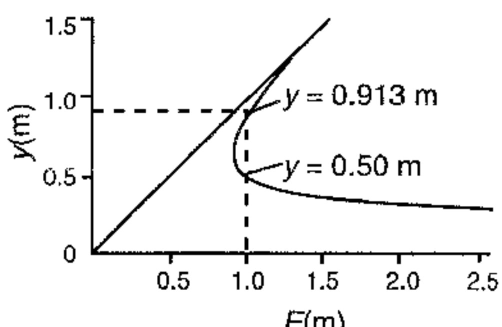
Part f of problem: Slopes for given *y* and alternate depth

y(m)	R(m)	S	
0.50	0.2649	0.0140	S for given y
0.913	0.3630	0.0021	S for alternate depth

Problem 14.40 Procedure: Refer to Table 14.2 for geometry of a partially full circular pipe.

a) For given Q, D, and y: Compute θ, A, T using equations in Table 14.2.
Compute $N_F = v/\sqrt{gy_h} = Q\left(A\sqrt{gy_h}\right)$.
Iterate values of y until N_F = 1.000. See spreadsheet: y_c **= 0.658 m.**

b) Minimum specific energy: $E = y + v^2/2g = y + Q^2/(2gA^2)$
From spreadsheet, with $y = y_c$ = 0.658 m: E_{min} **= 0.924 m.**



c) Specific energy versus y:
See spreadsheet using equation in b).

d) Specific energy for y = 0.50 m:
E = 1.039 m from spreadsheet.
Iterate on y to find alternate depth for which
E = 0.1039 m.
See spreadsheet: y_{alt} **= 0.913 m.**

e) Velocity = $v = Q/A$, $N_F = v/\sqrt{gy_h} = Q/\left(A\sqrt{gy_h}\right)$. See spreadsheet

For y = 0.50 m: v = 3.251 m/s, N_F = 1.690. Supercritical
For y = 0.913 m: v = 1.571 m/s, N_F = 0.528. Subcritical.

f) Compute $WP = \theta D/2$ (See Table 14.2). Compute $R = A/WP$.

Compute S: $\left[\frac{nQ}{AR^{2/3}}\right]^2$

See spreadsheet: For $y = 0.50$ m, $S = 0.0140$. For $y = 0.913$ m, $S = 0.0021$.

14.41 **Triangular channel**

$z =$ 1.5 $n = 0.022$

$Q =$ 0.68 ft³/s

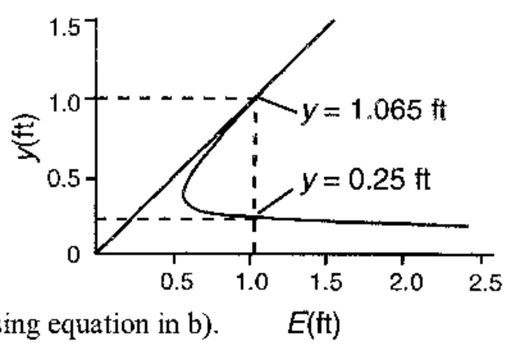
y(ft)	A(ft²)	V(ft/s)	T(ft)	y_h(ft)	N_F	E(ft)	
0.20	0.060		0.60	0.100	6.316	2.194	
0.25	**0.094**	**7.253**	**0.75**	**0.125**	**3.615**	**1.067**	**Given y**
0.30	0.135		0.90	0.150	2.292	0.694	
0.40	0.240		1.20	0.200	1.116	0.525	
0.418	**0.262**	**2.594**	**1.254**	**0.209**	**1.000**	**0.523**	**Critical depth**
0.50	0.375		1.50	0.250	0.639	0.551	
0.60	0.540		1.80	0.300	0.405	0.625	
0.70	0.735		2.10	0.350	0.276	0.713	
0.80	0.960		2.40	0.400	0.197	0.808	
0.90	1.215		2.70	0.450	0.147	0.905	
1.00	1.500		3.00	0.500	0.113	1.003	
1.065	**1.701**	**0.400**	**3.20**	**0.533**	**0.097**	**1.067**	**Alternate depth**
1.10	1.815		3.30	0.550	0.089	1.102	
1.20	2.160		3.60	0.600	0.072	1.202	
1.30	2.535		3.90	0.650	0.059	1.301	
1.40	2.940		4.20	0.700	0.049	1.401	
1.50	3.375		4.50	0.750	0.041	1.501	

Slopes at given depth and alternate depth

y(ft)	R(ft)	S	
0.25	0.10401	0.521	Slope for given depth
1.065	0.44307	0.000229	Slope for alternate depth

Problem 14.41 Procedure: Refer to Table 14.2 for geometry of a triangular channel.

a) For given Q, z, and y: Compute A, T using equations in Table 14.2.
Compute $N_F = v/\sqrt{gy_h} = Q/\left(A\sqrt{gy_h}\right)$.
Iterate values of y until $N_F = 1.000$.
See spreadsheet: $\mathbf{y_c = 0.418}$ **ft.**



b) Minimum specific energy:
$E = y + v^2/2g = y + Q^2/(2gA^2)$
From spreadsheet, with $y = y_c = 0.418$ ft:
$\mathbf{E_{min} = 0.523}$ **ft.**

c) Specific energy versus y: See spreadsheet using equation in b).

d) Specific energy for $y = 0.25$ ft: $E = 1.067$ ft from spreadsheet.
Iterate on y to find alternate depth for which $E = 0.1067$ ft.
See spreadsheet: $\mathbf{y_{alt} = 1.065}$ **ft.**

e) Velocity = $v = Q/A$, $N_F = v/\sqrt{gy_h} = Q/\left(A\sqrt{gy_h}\right)$. See spreadsheet

For y = 0.25 ft: v = 7.253 ft/s, N_F = 3.615. Supercritical
For y = 1.065 ft: v = 0.400 ft/s, N_F = 0.097. Subcritical.

f) Compute $WP = 2y\sqrt{1+z^2}$ (See Table 14.2). Compute $R = A/WP$.

Compute S: $\left[\dfrac{nQ}{AR^{2/3}}\right]^2$

See spreadsheet: For **y = 0.25 ft, S = 0.0521.** For **y = 1.065 ft, S = 0.000229.**

14.42 Trapezoidal channel

z = 0.75 n = 0.013
Q = 0.80 ft^3/s b = 3.000 ft

y(ft)	A(ft^2)	V(ft/s)	T(ft)	Y_h(ft)	N_F	E(ft)	
0.05	**0.152**	**5.267**	**3.08**	**0.049**	**4.177**	**0.481**	**Given y**
0.1	0.308	2.602	3.15	0.098	1.467	0.205	
0.1288	**0.399**	**2.006**	**3.19**	**0.125**	**1.000**	**0.191**	**Critical depth**
0.20	0.630	1.270	3.30	0.191	0.512	0.225	
0.25	0.797	1.004	3.38	0.236	0.364	0.266	
0.30	0.968	0.827	3.45	0.280	0.275	0.311	
0.40	1.320	0.606	3.60	0.367	0.176	0.406	
0.4770	**1.602**	**0.499**	**3.72**	**0.431**	**0.134**	**0.481**	**Alternate depth**
0.50	1.688	0.474	3.75	0.450	0.125	0.503	
0.60	2.070	0.386	3.90	0.531	0.093	0.602	
0.70	2.468	0.324	4.05	0.609	0.073	0.702	
0.80	2.880	0.278	4.20	0.686	0.059	0.801	
0.90	3.308	0.242	4.35	0.760	0.049	0.901	
1.00	3.750	0.213	4.50	0.833	0.041	1.001	
1.065	4.046	0.198	4.60	0.880	0.037	1.066	
1.10	4.208	0.190	4.65	0.905	0.035	1.101	
1.20	4.680	0.171	4.80	0.975	0.031	1.200	
1.30	5.168	0.155	4.95	1.044	0.027	1.300	
1.40	5.670	0.141	5.10	1.112	0.024	1.400	
1.50	6.188	0.129	5.25	1.179	0.021	1.500	

Slopes at given depth and alternate depth

y(ft)	R(ft)	S	
0.05	0.0486	0.264	Slope for given depth
0.477	0.3820	0.000152	Slope for alternate depth

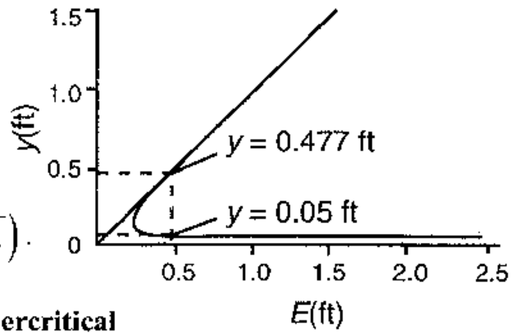
Problem 14.42 Procedure: Refer to Table 14.2 for geometry of a trapezoidal channel.

a) For given Q, b, z, and y: Compute A, T using equations in Table 14.2.
Compute $N_F = v/\sqrt{gy_h} = Q/\left(A\sqrt{gy_h}\right)$.
Iterate values of y until N_F = 1.000. See spreadsheet: **y_c = 0.1288 ft.**

b) Minimum specific energy: $E = y + v^2/2g = y + Q^2/(2gA^2)$
From spreadsheet, with $y = y_c$ = 0.1288 ft: **E_{min} = 0.191 ft.**

c) Specific energy versus y: See spreadsheet using equation in b).

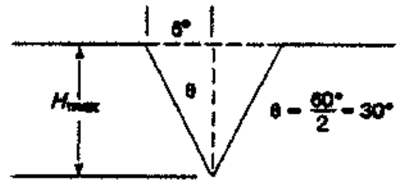
d) Specific energy for $y = 0.05$ ft: $E = 0.481$ ft from spreadsheet.
Iterate on y to find alternate depth for which $E = 0.481$ ft.
See spreadsheet: y_{alt} = **0.4770 ft.**



e) Velocity $= v = Q/A$, $N_F = v/\sqrt{gy_h} = Q/\left(A\sqrt{gy_h}\right)$.
See spreadsheet
For y = **0.05 ft: v = 5.267 ft/s, N_F = 4.177. Supercritical**
For y = **0.4770 ft: v = 0.499 ft/s, N_F = 0.134. Subcritical.**

f) Compute $WP = b + 2y\sqrt{1+z^2}$ (See Table 14.2). Compute $R = A/WP$.
Compute S: $\left[\frac{nQ}{AR^{2/3}}\right]^2$
See spreadsheet: For $y = 0.05$ ft, $S = 0.264$. For $y = 0.4770$ ft, $S = 0.000152$.

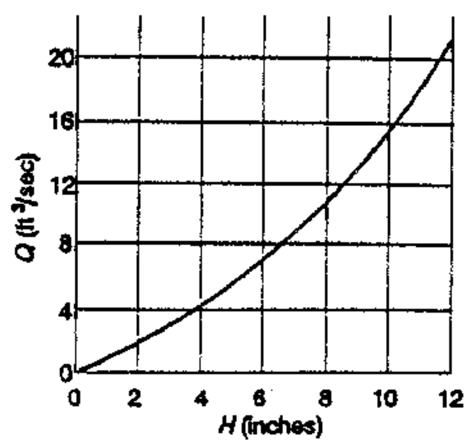
14.43 $H_{max} = 6$ in/tan 30° = 10.4 in × ft/12 in = 0.867 ft
$Q_{max} = 1.43\, H_{max}^{5/2} = (1.43)(0.867)^{5/2} =$ **1.00 ft^3/sec**



14.44 $H = 1.5$ ft; $H_c = 3$ ft; $Q = 15$ ft^3/sec; use Eq. (14.15)
$Q = [3.27 + 0.40(1.5/3.0)][L - 0.2(1.5)](1.5)^{3/2}$
$= (3.47)(L - 0.3)(1.838)$
$$L - 0.3 = \frac{Q}{(3.47)(1.838)} = \frac{15}{(3.47)(1.838)} = 2.36 \text{ ft}$$
$L = 2.36$ ft + 0.30 ft = **2.66 ft**

14.45 $(Q = 3.27 + 0.40H/H_c)LH^{3/2}$
$H_c = 2$ ft; $L = 6$ ft

H(in)	H(ft)	**Prob. (15.10)** Q(ft^3/sec)	**(15.11)** Q
0	0	0.00	0.00
2	.167	1.35	1.34
4	.333	3.84	3.80
6	.500	7.14	7.03
8	.667	11.10	10.90
10	.833	15.70	15.48
12	1.000	20.80	20.40

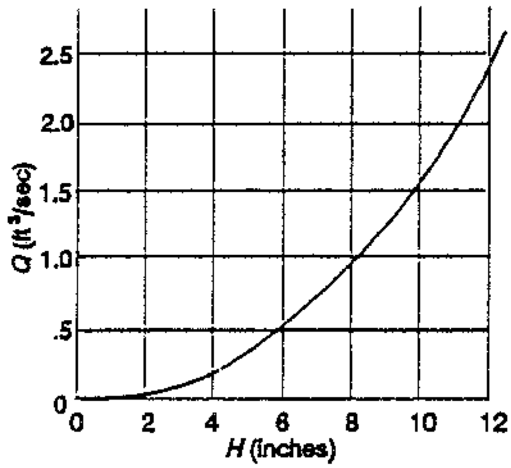


14.46 $Q = (3.27 + 0.40H/H_c(L - 0.2H)H^{3/2}$
Negligible difference on graph.

14.47 (a) $Q = (3.27 + 0.4H/H_c)LH^{3/2} = [3.27 + 0.4(1.5/4)](3)(1.5)^{3/2} =$ **18.8 ft^3/sec**
(b) $Q = (3.27 + 0.4H/H_c)(L - 0.2H)H^{3/2} = (3.42)(2.70)(1.5)^{3/2} =$ **16.95 ft^3/sec**
(c) $Q = 2.48H^{5/2} = (2.48)(1.5)^{5/2} =$ **6.84 ft^3/sec**

14.48 $Q = 2.48H^{5/2}$

H(in)	H(ft)	Q(ft^3/sec)
0	0	0
2	.167	.0283
4	.333	.159
6	.500	.439
8	.667	.900
10	.833	1.57
12	1.000	2.48



14.49 $Q = 3.07H^{1.53}$

$H^{1.53} = Q/3.07 \therefore H = (Q/3.07)^{1/1.53}$

Min $Q = 0.09$ ft^3/sec

$H = (0.09/3.07)^{1/1.53} = (0.0293)^{0.654}$

$=$ **0.10 ft**

Max $Q = 8.9$ ft^3/sec

$H = (8.9/3.07)^{1/1.53} = (2.90)^{0.654}$

$=$ **2.01 ft**

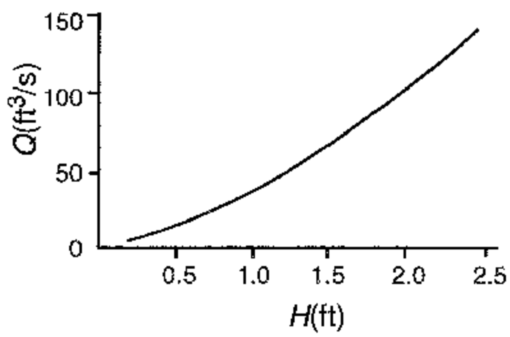
14.50 $L = 8.0$ ft; $Q_{min} = 3.5$ ft^3/s; $Q_{max} = 139.5$ ft^3/s

$Q = 4.00\,LH^n$; $H = [Q/4.00)L)^{1/n} = [Q/(4.00)(8.0)]^{1/1.61} = [Q/32]^{0.621}$

$Q_{min} = 3.5$ ft^3/s; $H = [3.5/32]^{0.621} =$ **0.253 ft**

$Q_{max} = 139.5$ ft^3/s; $H = [139.5/32]^{0.621} =$ **2.496 ft**

H(ft)	Q(ft^3/sec)
0.25	3.434
1.00	32.000
1.50	61.469
2.00	97.681
2.25	118.077
2.50	139.905



14.51 a) $Q = 50$ ft^3/s; $L = 4.0$ ft; $Q = 4.00\,LH^n$; $n = 1.58$

$H = [Q/(4.00)L]^{1/n} = [50/(4.00)(4.0)]^{1/1.58} = [3.125]^{0.633} =$ **2.06 ft**

b) $L = 10.0$ ft

$Q = (3.6875L + 2.5)H^{1.6} = 39.375H^{1.6}$

$$H = \left(\frac{Q}{39.375}\right)^{1/1.6} = \left(\frac{50}{39.375}\right)^{0.625} = \mathbf{1.155\ ft}$$

14.52 Trapezoidal channel—Long-throated flume – Design C: $H = 0.84$ ft; $Q = K_1(H + K_2)^n$

$K_1 = 16.180$; $K_2 = 0.035$; $n = 1.784$

$Q = 16.180[0.84 + 0.035]^{1.784} =$ **12.75 ft^3/s = Q**

14.53 Trapezoidal channel—Long-throated flume – Design B: $H = 0.65$ ft; $Q = K_1(H + K_2)^n$
$K_1 = 14.510$; $K_2 = 0.053$; $n = 1.855$
$Q = 14.510[0.65 + 0.053]^{1.855} = \mathbf{7.547\ ft^3/s = Q}$

14.54 Rectangular channel—Long-throated flume – Design A: $H = 0.35$ ft; $Q = b_cK_1(H + K_2)^n$
$b_c = 0.500$ ft; $K_1 = 3.996$; $K_2 = 0.000$; $n = 1.612$
$Q = (0.500)(3.996)[0.35 + 0.000]^{1.612} = \mathbf{0.368\ ft^3/s = Q}$

14.55 Rectangular channel—Long-throated flume – Design C: $H = 0.40$ ft; $Q = b_cK_1(H + K_2)^n$
$b_c = 1.500$ ft; $K_1 = 3.375$; $K_2 = 0.011$; $n = 1.625$
$Q = (1.500)(3.375)[0.40 + 0.011]^{1.625} = \mathbf{1.194\ ft^3/s = Q}$

14.56 Circular channel—Long-throated flume – Design B: $H = 0.25$ ft; $Q = D^{2.5}\,K_1\,(H/D + K_2)^n$
$D = 2.00$ ft; $K_1 = 3.780$; $K_2 = 0.000$; $n = 1.625$
$Q = (2.00)^{2.5}(3.780)[0.25/2.00 + 0.000]^{1.625} = \mathbf{0.729\ ft^3/s = Q}$

14.57 Circular channel—Long-throated flume – Design A: $H = 0.09$ ft; $Q = D^{2.5}\,K_1\,(H/D + K_2)^n$
$b_c = 1.00$ ft; $K_1 = 3.970$; $K_2 = 0.004$; $n = 1.689$
$Q = (1.00)^{2.5}(3.970)[0.09/1.00 + 0.004]^{1.689} = \mathbf{0.0732\ ft^3/s = Q}$

14.58 Rectangular channel—Long-throated flume – Design B: $Q = 1.25$ ft^3/s; Find H.
$Q = b_c\,K_1(H + K_2)^n$; $b_c =$ 1.00 ft; $K_1 = 3.696$; $K_2 = 0.004$; $n = 1.617$
Solving for H: $H = [Q/(b_cK_1)]^{1/n} - K_2 = [1.25/(1.0)(3.696)]^{1/1.617} - 0.004 = \mathbf{0.507\ ft = H}$

14.59 Circular channel—Long-throated flume – Design C: $Q = 6.80$ ft^3/s; Find H.
$Q = D^{2.5}\,K_1(H/D + K_2)^n$; $D = 3.000$ ft; $K_1 = 3.507$; $K_2 = 0.000$; $n = 1.573$
Solving for H: $H = D\{[Q/D^{2.5})(K_1)]^{1/n} - K_2\} = 3.0\{[6.80/(3.0^{2.5})(3.507)]^{1/1.573} - 0.00\}$
$= \mathbf{0.797\ ft = H}$

14.60 Select a long-throated flume for 30 gpm < Q < 500 gpm. Using 449 gpm = 1.0 ft^3/s,
0.0668 ft^3/s < Q < 1.114 ft^3/s; Select Circular channel; Design A; $Q = D^{2.5}\,K_1(H/D + K_2)^n$
$H = D\{[Q/D^{2.5})(K_1)]^{1/n} - K_2\}$; $D = 1.000$ ft; $K_1 = 3.970$; $K_2 = 0.004$; $n = 1.689$
For $Q = 0.0668$ ft^3/s: $H = -1.0\{[0.0668/(1.0^{2.5})(3.970)]^{1/1.689} - 0.004\} = \mathbf{0.0851\ ft = H}$
For $Q = 1.114$ ft^3/s: $H = -1.0\{[1.114/(1.0^{2.5})(3.970)]^{1/1.689} - 0.004\} = \mathbf{0.467\ ft = H}$

H	Q(ft^3/s)	Q(gpm)
0.10	0.087	39.06
0.20	0.271	121.7
0.30	0.531	238.4
0.40	0.859	385.7

14.61 Given 50 m^3/h < Q , 180 m^3/h; Convert to ft^3/s; 0.4907 ft^3/h < Q < 1.766 ft^3/h
Specify Rectangular channel long-throated flume, Design B.
Find H for each limiting flow rate.
$Q = b_c K_1(H + K_2)^n$; $b_c = 1.00$ ft; $K_1 = 3.696$; $K_2 = 0.004$; $n = 1.617$
Solving for H: $H_{min} = [Q/(b_cK_1)]^{1/n} - K_2 = [0.4907/(1.0)(3.696)]^{1/1.617} - 0.004$
= **0.2829 ft = H_{min}**
Converting to m: **H_{min} = 0.0863 m for Q = 50 m^3/h**
$H_{max} = [Q/b_cK_1)]^{1/n} - K_2$ = [1.766/(1.0)(3.696)1/1.617 – 0.004 = 0.629 ft = H_{max}
Converting to m: **H_{max} = 0.1917 m for Q = 180 m^3/h**

H(m)	H(ft)	Q(ft^3/s)	Q(m^3/h)
0.100	0.328	0.622	63.38
0.125	0.410	0.888	90.49
0.150	0.492	1.190	121.3
0.175	0.524	1.524	155.3